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A mean-field game model of electricity market dynamics with environmental policies

Alicia Bassière, Roxana Dumitrescu, Peter Tankov

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Introduction

- The energy transition requires an increasing share of renewable generation and the closure of coal-fired power plants.
- Recent crises have highlighted the risk of relying more on gas power plants in the meantime.
- Modeling the investment dynamics of power plants is challenging because producers invest in power plants to earn revenues on a market that is dynamic with numerous players, involving uncertainties in both costs and production.
- Environmental policies further complicate market dynamics

Research Question

How can entry and exit dynamics in the electricity market be modeled, considering strategic behavior, uncertainty and environmental policies?

Main Contributions

This paper, originally published in Bassière, Dumitrescu, and Tankov 2024

- Develops a long-term model for the dynamics of the electricity industry that describes the energy transition
- Introduces strategic interactions under cost and production uncertainty, accounting for agent heterogeneity, construction lifetime, and endogenous fuel prices
- Proves the existence of equilibrium with the uniqueness of electricity and fuel prices
- Demonstrates the equivalence with the mean-field control central planner counterpart of the problem
- Analyzes the dynamics of entry and exit with environmental policies.

Related Literature

- Electricity market models can be classified into three categories (Ventosa et al. 2005):
  (1) Market equilibrium models: tractable equilibrium concept but with simplifying assumptions (e.g., static models, homogeneity of agents)
  (2) Optimization models: engineering models representing large power systems lacking clear strategic interaction representation
  (3) Simulation models: simulate the behavior of large power systems over time, difficult to compute and interpret

- Mean-Field Game models: offer a dynamic equilibrium concept for many players with a tractable solution, relaxing assumptions like agent homogeneity, and allowing for uncertainty and endogenous fuel prices.

A Mean-Field Game Model for Entry/Exit on the electricity market

The Agents

Each electricity producer j uses a technology of type i, from categories:

- **Conventional Power Plants**:
  - Operate one power plant, bidding a fraction ξ of this capacity on the market.
  - The production marginal cost function is given by:

$$C_t^{ij}(\xi) = \underbrace{c^i(\xi)}_{\text{Operating cost}} + \underbrace{f_i P_t^{k(i)}}_{\text{Fuel cost}} + \underbrace{f_i e_{k(i)} P_t^C}_{\text{Carbon cost}} + \underbrace{Z_t^{ij}}_{\text{Random cost (CIR)}} \tag{1}$$

- **Renewable Power Plants**:
  - Characterized by a random production capacity factor S_t^{ij} (Jacobi)
  - Bid their entire possible production on the market.
- **Baseload Power Plants**:
  - Aggregated supply function, unaffected by the market dynamics

Agents random component

Price Formation

Electricity Price:

- Agents offer electricity quantities to the market to meet an exogenous demand.
- Conventional producers choose a fraction ξ to maximize revenue, while renewable producers offer their full production.
- If the market fails (e.g., insufficient supply), the electricity price is capped at P^*.

Fuel Price:

- Each fuel type has an exogenous supply function.
- The fuel price is determined by matching this supply function to the total fuel consumption for electricity production with associated technologies

Price equations

Entry on the Market

- Potential producers aim to optimize their market entry (τ_1) and exit (τ_2) times:
  - Maximize expected revenues and minimize entry costs, conditional on their chosen entry and exit times.
- Conventional producers already in the market evaluate the optimal exit time (τ_2):
  - Maximize expected market revenues and scrapping value, conditional on the exit time.
- The problem includes considerations of fixed costs, construction time, plant lifetime, dynamic investment costs for technology i

Entry on the Market

- Potential producers aim to optimize their market entry (τ_1) and exit (τ_2) times in order to:
  - Maximize expected revenues and minimize entry costs, conditional on their chosen entry and exit times.
- Conventional producers already in the market evaluate the optimal exit time (τ_2) to:
  - Maximize expected market revenues and scrapping value, conditional on the exit time.
- The problem includes considerations of fixed costs, construction time, asset lifetime, dynamic investment costs for technology i
- The **central planner** controls market entries and exits to maximize the total revenues of all agents, minus consumer electricity costs.
- Proven equivalence with the mean-field game problem.

Nash Equilibrium

- Classical Nash Equilibrium: Agent j chooses strategies (τ_1^j, τ_2^j) without incentive to deviate, considering others' strategies
- **Challenging to compute for numerous players!**

Nash Equilibrium for perfect competition

- Classical Nash Equilibrium: Agent j chooses strategies (τ_1^j, τ_2^j) without incentive to deviate, considering others' strategies
- **Challenging to compute for numerous players!**
- Mean-Field Theory: Replace class of agent j by an infinite population of agents of type j, described by a distribution $m_t^j(da, dx)$ of ages and costs
- Mean-Field Nash Equilibrium: A representative agent of class j has no incentive to deviate given distributions $m_t^{-j}(da, dx)$

▸ Mean-Field formulation

Linear programming approach

- Each agent maximises its expected gains as a linear function of the occupation measure of the population in the market, allowing to use the linear programming approach
- Addition of linear constraints on the measures to respect the stochastic process dynamics for cost functions and renewable capacity factor

- MF Nash equilibrium a sequence of entry/exit measures and price functions such that:
  1. For each $i = 1, ..., N$, measures maximize the conventional producers program
  2. For each $i = N+1, ..., \overline{N}$, measures maximize the renewable producers program
  3. For each t, the price vector is the solution of the system of demand matching supply

▸ Formal definition of Nash equilibrium ▸ Numerical resolution

Numerical illustration

Calibration

- German data over 25 years starting from 2019, with 24 representative hours per year (peak/off-peak) and **2.3%** annual growth
- Coal, Gas, Wind for entry and exit
- **3%** annual reduction in renewable investment costs (learning effect)

Environmental Policies:

- Carbon tax: from **20€** to **200€** by 2045
- Renewable production subsidies: from **60€** in 2019 to **0€** by 2045

Benchmark scenario

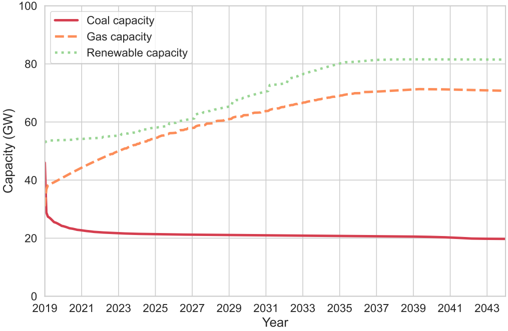

Figure 1: Evolution of off-peak supply

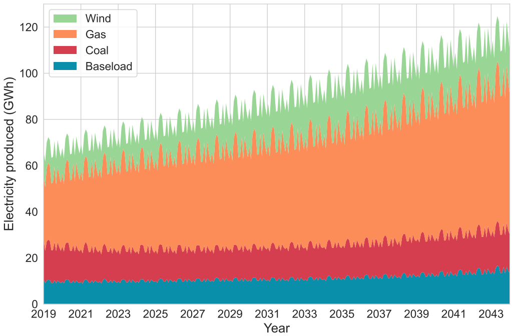

Figure 2: Evolution of peak supply

Carbon Tax: Massive renewable entry and coal phase-out

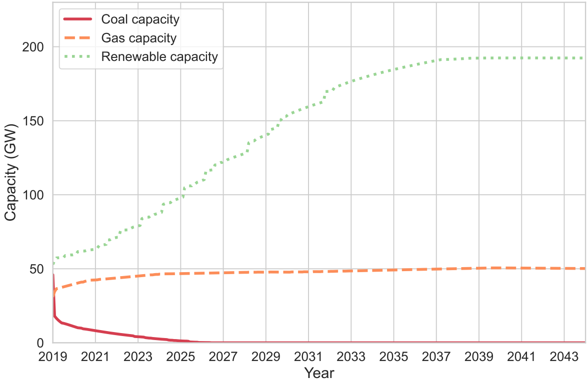

Figure 3: Evolution of capacities

Carbon Tax: ... but gas production rises in the long run

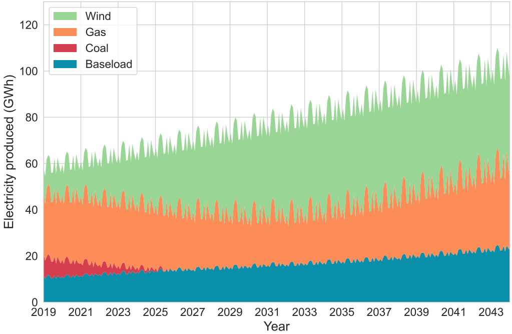

Figure 4: Evolution of off-peak supply

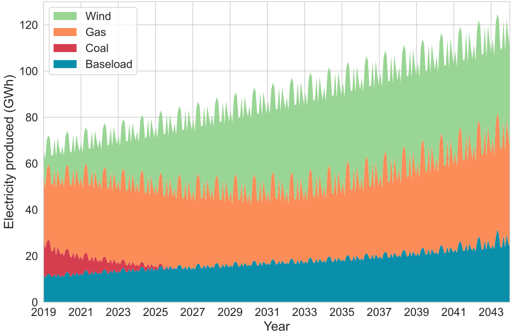

Figure 5: Evolution of peak supply

Carbon Tax: ... with strong electricity prices

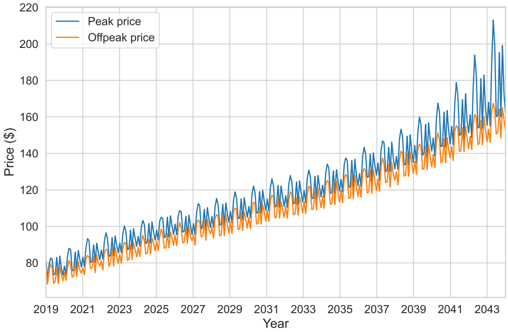

Figure 6: Evolution of electricity prices

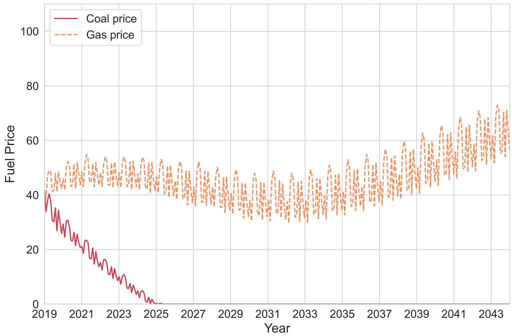

Figure 7: Evolution of fuel prices

RES Subsidy: Faster renewable penetration

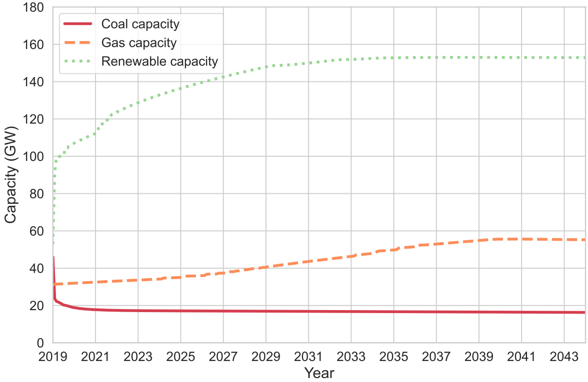

Figure 8: Evolution of capacities

RES Subsidy: ...but slowed in the long run

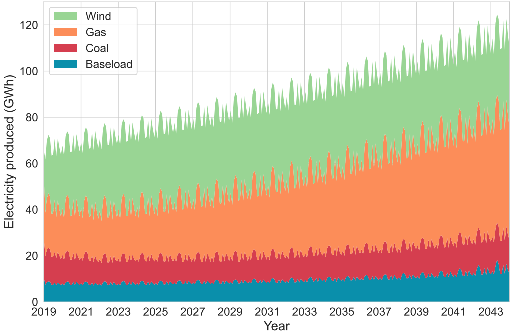

Figure 9: Evolution of off-peak supply

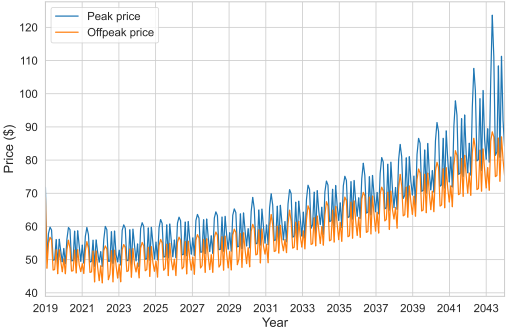

Figure 10: Evolution of peak supply

Discussion

- Gas plants remain competitive due to low costs, lower emissions, and predictable production
- Carbon tax phases out coal and promotes renewables, but raises electricity prices and does not defer gas entries
- Renewable subsidies accelerate renewables, but only with a subsidy threshold and do not push coal to phase-out
- Future work: central planning will allow the introduction of other environmental policies, like carbon emissions societal constraints

Conclusion

- We developed a mean field game approach for electricity market dynamics, incorporating endogenous fuel prices and many technical features
- We proved the existence of an equilibrium with unique electricity and fuel prices
- Without environmental policies, gas plants meet demand
- The carbon tax accelerates the coal phase-out by 2030, promotes renewable energy, but leads to higher long-term electricity prices.
- Renewable subsidies initially support wind energy but become ineffective beyond a certain threshold and fail to push coal out of the market, leading to more carbon emissions.

Thank you for your attention

Contact: alicia.bassiere@centralesupelec.fr

Website:

References I

Bassière, Alicia, Roxana Dumitrescu, and Peter Tankov (2024). “A mean-field game model of electricity market dynamics”. In: *Quantitative Energy Finance: Recent Trends and Developments*. Springer, pp. 181–219.

Ventosa, Mariano et al. (2005). “Electricity market modeling trends”. In: *Energy policy* 33.7, pp. 897–913.

Pricing equations

$$\left(D_t^p - R_t\right)^+ = F_0\left(P_t^p\right) + \sum_{k=1} F_t^k\left(P_t^p, P_t^k\right), \tag{2}$$

$$\text{or } \left(D_t^p - R_t\right)^+ > F_0\left(P_t^p\right) + \sum_{t}^{K} F_t^k\left(P_t^p, P_t^k\right) \quad \text{and} \quad P_t^p = P^* \tag{3}$$

Fuel and price equations

- Fuel price solving:

$$c_p \Psi_t^k \left(P_t^p, P_t^k\right) + c_{op} \Psi_t^k \left(P_t^{op}, P_t^k\right) = \Phi_k \left(P_t^k\right) \tag{4}$$

- Fuel consumption:

$$\Psi_t^k \left(P^E, P^k\right) = \sum_{i:k(i)=k} \sum_{j=1}^{N_i} \lambda_i \left(t - \tau_1^{ij}\right) \mathbf{1}_{\tau_2^{ij} > t} f_i Q_{ij} F_i \left(P^E - f_i e_{k(i)} P^C - f_i P^k - Z_t^{ij}\right), \tag{5}$$

▸ Price formation

Agents production processes

- Renewable capacity factor for agent j with technology i:

$$dS_t^{ij} = \overline{k}^i(\overline{\theta}^i - S_t^{ij})dt + \overline{\delta}^i\sqrt{S_t^{ij}(1 - S_t^{ij})}dW_t^{ij}, \quad S_0^{ij} = \overline{s}_{ij} \tag{6}$$

- Random cost component for agent j with technology i:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i\sqrt{Z_t^{ij}}dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{7}$$

The agents

Agents maximization programs (I)

- Conventional producers instantenous gain function:

$$\int_0^{\xi^*} \left(p - C_t^{ij}(\xi)\right) d\xi = G_i \left(p - e_{k(i)} P_t^C + P_t^{k(i)} + Z_t^{ij}\right) \tag{8}$$

- Conventional cost function for agent *j* with technology *i*:

$$\mathbb{E}\left[\underbrace{\int_{\tau_1}^{\tau_2} e^{-\rho t} \lambda_i(t-\tau_1)\left(G_i\left(P_t - f_i e_{k(i)} P_t^C - f_i P_t^{k(i)} - Z_t^{ij}\right) - \kappa_i\right) dt}_{\text{Market gains}} - \underbrace{K_i e^{-(\rho+\gamma_i)\tau_1}}_{\text{Entry cost}} + \underbrace{\widetilde{K}_i e^{-(\rho+\gamma_i)\tau_2}}_{\text{Exit scrap. value}}\right]$$

Game of entry and exit

Agents maximization programs (II)

- Renewable supply function for agent *j* with technology *i*:

$$\mathbb{E}\left[\int_{\tau_1}^{\tau_2} \underbrace{e^{-\rho t}\lambda_i\left(t-\tau_1\right)\left(P_t S_t^i-\kappa_i\right)dt}_{\text{Market gains}} - \underbrace{K_i e^{-(\rho+\gamma_i)\tau_1}}_{\text{Entry cost}} + \underbrace{\widetilde{K}_i e^{-(\rho+\gamma_i)\tau_2}}_{\text{Exit scrap. value}}\right] \tag{9}$$

▸ Game setting

Central planner maximisation programm

$$
\begin{aligned}
\max_{(\tau_1,\tau_2)\in[0,T]^{M+M}} \Bigg\{ & \sum_{i=0}^{N} \mathbb{E}\Bigg[\int_{\tau_{1,i}}^{\tau_{2,i}} \underbrace{e^{-\rho t}\lambda_i(t-\tau_{1,i})\big(G_i(\cdot)-\kappa_i\big)dt - K_i e^{-(\rho+\gamma_i)\tau_{1,i}} + \widetilde{K}_i e^{-(\rho+\gamma_i)\tau_{2,i}}}_{\text{Conventional gains}} \Bigg] \\
& + \sum_{j=0}^{M-N} \mathbb{E}\Bigg[\int_{\tau_{1,j}}^{\tau_{2,j}} \underbrace{e^{-\rho t}\lambda_j(t-\tau_{1,j})\big(P_t S_t^j - \kappa_j\big)dt - K_j e^{-(\rho+\gamma_j)\tau_{1,j}} + \widetilde{K}_j e^{-(\rho+\gamma_j)\tau_{2,j}} \Bigg]}_{\text{Renewable gains}} \\
& + \Bigg[\int_{\tau_{1,j}}^{\tau_{2,j}} \underbrace{e^{-\rho t}(G_0(P_t)}_{\text{Baseline gains}} - \underbrace{P_t D_t}_{\text{Consumer costs}} \Big) + \sum_{k=1}^{K} \underbrace{\bar{\phi}_k(P_t^k)}_{\text{Fuel producer gains}} \Bigg] \Bigg\}.
\end{aligned}
\tag{10}
$$

▸ Game Setting

Infinitesimal Generator: Conventional

- Conventional cost function process:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i \sqrt{Z_t^{ij}} dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{11}$$

- Associated Infinitesimal generator for a C^2 u function:

$$\mathcal{L}_{ij} u = k^i(\theta^i - z)\frac{\partial u}{\partial z} + \frac{1}{2}(\delta^i)^2 z \frac{\partial^2 f}{\partial z^2}$$

▸ The agents

Infinitesimal Generator: Renewable

- Conventional cost function process:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i \sqrt{Z_t^{ij}} dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{12}$$

- Associated Infinitesimal generator for a C^2 u function:

$$\mathcal{L}_{ij} u = k^i(\theta^i - z)\frac{\partial u}{\partial s} + \frac{1}{2}(\delta^i)s(1-s)\frac{\partial^2 u}{\partial s^2}$$

▸ Constraints

Fuel and price equations

- Fuel price solving:

$$c_p \Psi_t^k \left(P_t^p, P_t^k\right) + c_{op} \Psi_t^k \left(P_t^{op}, P_t^k\right) = \Phi_k \left(P_t^k\right) \tag{13}$$

- Fuel consumption:

$$\Psi_t^k \left(P^E, P^k\right) = \sum_{i:k(i)=k} \sum_{j=1}^{N_i} \lambda_i \left(t - \tau_1^{ij}\right) \mathbf{1}_{\tau_2^{ij} > t} f_i Q_{ij} F_i \left(P^E - f_i e_{k(i)} P^C - f_i P^k - Z_t^{ij}\right), \tag{14}$$

Agents

Introduction of measures

- 2 classes of population for agent of type i
  - Class $\hat{C}_i$: plants which the decision to build has not been taken yet
  - Class C_i: plants under construction or operational
- **Occupation Measure** ($m_i(t)$)
  - **Purpose**: Represents the distribution of active agents over their state space at any given time
- **Entry Measure** (ν_i)
  - **Purpose**: Captures the rate and conditions of new market entrants over time
- **Exit Measure** (μ_i)
  - **Purpose**: Quantifies the rate at which agents withdraw from the market

Mean-Field formulation I

$$m_i^t(da, dx) = \int_{A \times O_i} \nu_0^i(da', dx') \mathbb{E}\left[\delta(a' + t, Z_t^i)(da, dx)\right] \tag{15}$$

$$\mu_i(dt, da, dx) = \int_{A \times O_i} \nu_0^i(da', dx') \mathbb{E}\left[\delta(\tau_2^i, \tau_2^i + a', Z_{\tau_2}^i)(dt, da, dx)\right] \tag{16}$$

$$\nu_i(dt, da, dx) = \nu_0^i(da, dx)\delta_0(dt) + \hat{\mu}_i(dt, dx)\delta_0(da) \tag{17}$$

Mean-Field formulation II

$$\hat{\mu}_i(dt, dx) = \int_{O_i} \hat{\nu}_0^i(dx')\mathbb{E}\left[\delta(\tau_1^i, Z_{\tau_1}^i)(dt, dx)\right] \tag{18}$$

$$\hat{m}_i^t(dx) = \int_{O_i} \hat{\nu}_0^i(dx')\mathbb{E}\left[\delta(Z_t^i)(dx)\right] \tag{19}$$

$$\hat{\nu}_i(dt, dx) = \hat{\nu}_0^i(dx)\delta_0(dt) \tag{20}$$

▸ Linear programming

MFG equations: price equations system

- Conventional supply function:

$$F_t^k\left(P^E, P^k\right) = \sum_{i:k(i)=k} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a)F_i\left(P^E - f_i e_k P^C - f_i P^k - x\right) \tag{21}$$

- Fuel consumption is therefore:

$$\Psi_t^k\left(P^E, P^k\right) = \sum_{i:k(i)=k} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a)f_i F_i\left(P^E - f_i e_k P^C - f_i P^k - x\right) \tag{22}$$

- Renewable supply function:

$$R_t = \sum_{i:k(i)=k}^{N+\overline{N}} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a) \tag{23}$$

MFG equations: optimization functionals

- Conventional gain function:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da,dx)e^{-\rho t}\lambda_i(a)\left(c_p G_i\left(P_t^p - f_i e_{k(i)}P_t^C - f_i P_t^{k(i)} - x\right)\right.$$
$$\left. + c_{op} G_i\left(P_t^{op} - f_i e_{k(i)}P_t^C - f_i P_t^{k(i)} - x\right) - \kappa_i\right) dt$$
$$-K_i \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \hat{\mu}^i(dt,da,dx)e^{-(\rho+\gamma_i)t} + \widetilde{K}_i \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \mu^i(dt,da,dx)e^{-(\rho+\gamma_i)t}$$

MFG equations: optimization functionals

- Renewable gain function:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da,dx)e^{-\rho t}\lambda_i(a)\left(\left(c_p P_t^p + c_{op}P_t^{op}\right)x - \kappa_i\right)dt$$
$$-K_i\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i}\hat{\mu}^i(dt,da,dx)e^{-(\rho+\gamma_i)t} + \widetilde{K}_i\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i}\mu^i(dt,da,dx)e^{-(\rho+\gamma_i)t}$$

Linear programming

MFG equations: constraints I

$$\int_{[0,T]\times A\times O_i} u(t,a,x)\nu_i(dt,da,dx) + \int_{[0,T]\times A\times O_i} \left(\frac{\partial u}{\partial t} + L_i u\right) m_i^t(da,dx)dt$$
$$= \int_{[0,T]\times A\times O_i} u(t,a,x)\mu_i(dt,da,dx)$$

$$\int_{[0,T]\times O_i} u(t,x)\hat{\nu}_i(dt,dx) + \int_{[0,T]\times O_i} \left(\frac{\partial \hat{u}}{\partial t} + \hat{L}_i u\right) \hat{m}_i{}^t(dx)dt$$
$$= \int_{[0,T]\times O_i} u(t,x)\hat{\mu}_i(dt,dx)$$

MFG equations: constraints II

$$\hat{\nu}_i(dt, dx) = \hat{\nu}_0^i(dx)\delta_0(dt) \tag{24}$$

$$\nu_i(dt, da, dx) = \nu_0^i(da, dx)\delta_0(dt) + \hat{\mu}_i(dt, dx)\delta_0(da) \tag{25}$$

▸ Linear programming

Nash equilibrium equations

- Denote $\mathcal{R}_i(\hat{\nu}_0^i, \nu_0^i)$ the class of n-uplets:

$$\left(\hat{\mu}^i, \left(\hat{m}_t^i\right)_{0\leq t\leq T}, \mu^i, \left(m_t^i\right)_{0\leq t\leq T}\right) \in \mathcal{M}_i \times \mathcal{V}_i \times \mathcal{M}_i \times \mathcal{V}_i \tag{26}$$

with for all $u \in C_b^{1,2,2}\left([0,T] \times \mathcal{A} \times \overline{\mathcal{O}}_i\right)$ satisfies

Nash equilibrium equations

The class satisfies the constraints:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\nu^i(dt,da,dx) + \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \left\{\frac{\partial u}{\partial t} + \mathcal{L}_i u\right\} m_t^i(da,dx)dt$$
$$= \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\mu^i(dt,da,dx) (11)$$

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\hat{\nu}^i(dt,da,dx) + \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \left\{\frac{\partial u}{\partial t} + \mathcal{L}_i u\right\} \hat{m}_t^i(da,dx)dt$$
$$= \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\hat{\mu}^i(dt,da,dx)$$

$$\hat{\nu}_i(dt,dx) = \hat{\nu}_0^i(dx)\delta_0(dt)$$

$$\nu_i(dt,da,dx) = \nu_0^i(da,dx)\delta_0(dt) + \hat{\mu}_i(dt,dx)\delta_0(da)$$

Nash equilibrium theorem

- Assume that the initial measures satisfy

$$\int_{\mathcal{O}_i} \ln(1+|x|)\hat{\nu}_0^i(dx) + \int_{\mathcal{A}\times\mathcal{O}_i} \ln(1+|x|)\nu_0^i(da,dx) < \infty. \tag{27}$$

- Assume that the peak demand D^p, the off-peak demand D^{op} and the carbon price P^C have finite variation on $[0, T]$
- Existence of a relaxed Nash equilibrium

▸ Nash equilibrium

Mean-Field formulation of the central planner

$$J(\boldsymbol{m}) = \int_0^T e^{-\rho t} \min_{P \in [0,P^*]^2 \times \mathbb{R}_+^K} G_t(\boldsymbol{m}, P) dt - \sum_{i=1}^{N+\overline{N}} \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx) e^{-\rho t} \lambda_i(a) \kappa_i$$
$$- \sum_{i=1}^{N+\overline{N}} K_i \int_{[0,T]\times\overline{\mathcal{O}}_i} \hat{\mu}^i(dt, dx) e^{-(\rho+\gamma_i)} + \sum_{i=1}^{N+\overline{N}} \widetilde{K}_i \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \mu^i(dt, da, dx) e^{-(\rho+\gamma_i)t}. \quad (28)$$

▸ Nash equilibrium

Numerical resolution: the fictitious play algorithm

- For each group of technologies:
  1. Initialize with a "guess" on the strategy
  2. Describe optimal strategies for a representative agent as a function of the population distribution
  3. Population distribution update in case of strategy profitability
  4. Repeat until stationarity of the strategy (no more profitable deviation)
     $\rightarrow$ **Satisfactory approximation of Nash Equilibrium**

▸ Numerical algorithm

Numerical resolution : The fictitious play algorithm

1. Initialization:

$$\left(\hat{\mu}^{i,0}, \left(\hat{m}_t^{i,0}\right)_{0\leq t\leq T}, \mu^{i,0}, \left(m_t^{i,0}\right)_{0\leq t\leq T}\right) \in \mathcal{R}_i, \quad i = 1, \dots, N + \bar{N}$$

2. Compute prices $(P_{tp}, P_{top}, P_{t1}...P_{tK})_{0\leq t\leq T}$
3. Optimize the agents program to get best responses
4. Measures update:

$$\begin{aligned}\left(\hat{\mu}^{i,j}, \left(\hat{m}_t^{i,j}\right)_{0\leq t\leq T}, \mu^{i,j}, \left(m_t^{i,j}\right)_{0\leq t\leq T}\right) &= \varepsilon_j \left(\hat{\bar{\mu}}^{i,j}, \left(\hat{\bar{m}}_t^{i,j}\right)_{0\leq t\leq T}, \bar{\mu}^{i,j}, \left(\bar{m}_t^{i,j}\right)_{0\leq t\leq T}\right) \\ &+ (1-\varepsilon_j) \left(\hat{\mu}^{i,j-1}, \left(\hat{m}_t^{i,j-1}\right)_{0\leq t\leq T}, \mu^{i,j-1}, \left(m_t^{i,j-1}\right)_{0\leq t\leq T}\right)\end{aligned}$$

▸ Linear programming