

Can investors curb greenwashing?

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Workshop: Energy, mathematics, and theoretical challenges

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Greenwashing: a likely widespread practice

Greenwashing: *The practice by which companies claim they are doing more for the environment than they actually are.* (European Commission).



EN English

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Screening of websites for 'greenwashing': half of green claims lack evidence

→ Annual screening of company websites (European Commission, 2021): In **42% of cases**, the authorities “had reason to believe that the **[company’s] claim may be false or deceptive.**”

Why would companies greenwash?

Companies have (i) the incentive and (ii) the ability to overstate their environmental value.

Incentive to greenwash:

1. In equilibrium, environmentally well-rated companies benefit from lower costs of capital (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022).

Ability to greenwash:

2. Companies can benefit from information asymmetry about their true environmental values (Barbalau and Zeni, 2023) and communicate in an ambiguous manner (Fabrizio and Kim, 2019).
3. The reliability of environmental scores is questionable (Berg et al., 2022):
 - ▶ companies' environmental footprints are challenging to measure accurately,
 - ▶ measurement methods are not standardized.

Greenwashing: a major issue

For investors: major obstacle to

- (i) environment-related risk assessment;
- (ii) environmental impact of investments.

Questions:

- What are the incentives for companies to greenwash?
- When do companies use environmental communication to greenwash?
- What role can investors play in influencing greenwashing practices?

Challenge: Modeling a **strategy** that is (i) **complex** (two controls, information asymmetry), (ii) **dynamic**, and (iii) involves **uncertainty** (on the score and the controversies that arise).

What we do

1. **We build a dynamic asset pricing equilibrium model** with
 - ▶ **Information asymmetry** about companies' **environmental value**;
 - ▶ n heterogeneous **companies** which can (i) **communicate** and (ii) **reduce their emissions** to influence their **environmental score**;
 - ▶ A **representative investor** (i) with **pro-environmental preferences** and (ii) who can **penalize** revealed environmental **misrating** (through the occurrence of controversies).

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- ▶ equilibrium expected returns;
- ▶ companies' optimal environmental strategy and **greenwashing strategy**;
- ▶ and show how it is **impacted** by investor's **green preferences** and **penalty**.

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 - ▶ equilibrium expected returns;
 - ▶ companies' optimal environmental strategy and **greenwashing strategy**;
 - ▶ and show how it is **impacted** by investor's **green preferences** and **penalty**.
3. **We validate empirically** the environmental communication dynamics of green companies.

What we find

1. **Companies** (i) **greenwash to inflate their environmental score** above their fundamental environmental value (because of investors' pro-environmental preferences) (ii) **up to a certain level** of discrepancy (because of the investor's penalty), (iii) **under certain conditions** (high inform. asymmetry; low relative marg. unit cost of com. vs. abatement).

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 - (i) regulations strengthening **transparency**;
 - (ii) support for environmental **technological innovation**.

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3. **Policymakers** can also **curb greenwashing** and **increase abatement**:
 - (i) regulations strengthening **transparency**;
 - (ii) support for environmental **technological innovation**.
4. **Empirical evidence** suggests that **companies greenwash** (especially green ones) through their environmental communication **depending on the recent change in their environmental score**.

Contributions to the literature

- **Greenwashing and environmental disclosure:** Duflo et al. (2013); Duchin et al. (2023); Hoepner et al. (2017); Bingler et al. (2022, 2023) and Flammer (2021); Ilhan et al. (2023); Berg et al. (2022, 2021); Chen (2024).
 - ▶ **First theoretical paper linking greenwashing to investment decisions.**
- **Sustainable asset pricing:** Pástor et al. (2021); Pedersen et al. (2021); Zerbib (2022); Bolton and Kacperczyk (2021); De Angelis et al. (2023); Pástor et al. (2022); Zerbib (2022); Cheng et al. (2023); Avramov et al. (2022); Sauzet and Zerbib (2022); Berk and van Binsbergen (2021); Goldstein et al. (2022); Pástor et al. (2022); Ardia et al. (2023); Van der Beck (2023).
 - ▶ **Correction for greenwashing in addition to green premium on expected returns.**
- **Asset pricing and information asymmetry:** Grossman and Stiglitz (1980); Admati and Pfleiderer (1986); Hughes (1986); Easley and O'hara (2004); Lambert et al. (2012).
 - ▶ **Asset pricing model with random revelation times.**
- **Impact investing:** De Angelis et al. (2023); Hartzmark and Shue (2023); Favilukis et al. (2023); Green and Roth (2024); Oehmke and Opp (2024); Green and Roth (2024); Landier and Lovo (2023); Edmans et al. (2023); Barber et al. (2021); Bonnefon et al. (2022); Heeb et al. (2023).
 - ▶ **Double positive impact of investors: curb greenwashing & foster abatement.**

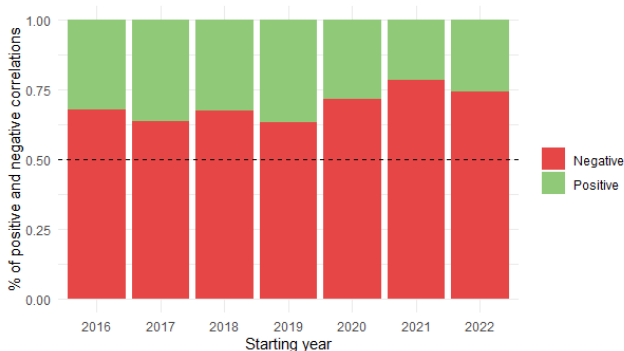
Outline

- 1 Empirical motivation
- 2 A dynamic equilibrium model with corporate greenwashing
- 3 Optimal greenwashing and investor's impact
- 4 Empirical evidence
- 5 Introducing interaction between companies

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Environmental communication used to correct the environmental score?



Observation 2: 63% to 78% of companies show a **negative correlation** between variations in their **environmental reputation score** (= monthly “reputation flow”) and their **previous month’s environmental score**.

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Market setting

Probability space $(\Omega, \mathbb{F} = (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ with **infinite** time horizon.

Assets:

- 1 risk-free asset with zero interest rate
- n **firms** issuing stocks at quantity normalized to 1, indexed by i

Price process of the risky assets, $S \in \mathbb{R}^n$:

$$dS_t = \mu_t dt + \sigma dB_t,$$

- $\mu_t \in \mathbb{R}^n$ vector of expected returns, determined at equilibrium
- $\sigma \in \mathbb{R}^{n \times n}$ exogenously specified constant volatility matrix
- $B \in \mathbb{R}^n$ a.s. a brownian motion

Environmental score

Fundamental environmental value of company i :

$$dV_t^i = \underbrace{r_t^i dt}_{\text{Abatement effect}}, \quad V_0^i = p^i,$$

with r^i the **emissions reduction (or abatement) effort** of company i .

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Proxy for this value:

Environmental score of company i : $E_0^i = q^i$,

$$dE_t^i = \underbrace{a(V_t^i - E_t^i)dt}_{\text{Rating agency effect}} + \underbrace{b(V_t^i - E_t^i)dN_t^i}_{\text{Controversy effect}} + \underbrace{c_t^i dt}_{\text{Communication effect}} + \underbrace{z dW_t^i}_{\text{Measurement error}},$$

- c^i the **environmental communication effort** of company i
- N^i Poisson process, W^i brownian motion, independent from one another

Score for environmental misrating

Communication effort c^j

- allows the company to influence its score ($c > 0$, < 0 , or $= 0$)
- and can be **deceptive**.

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⇒ use of **controversies history** which reveal a portion $b \in [0, 1]$ of the ongoing misrating (through jumps of N^i).

Investor's score for environmental misrating:

$$dM_t^i = \underbrace{-\rho M_t^i dt}_{\text{Forgetting rate}} + \underbrace{(E_t^i - E_{t-}^i)^2 dN_t^i}_{\text{Square of misrating revealed by controversies}}, \quad M_0^i = u^i$$

Formal definition of greenwashing

Greenwashing is any green communication effort that aims at **creating** or **increasing** a **positive gap** between the **environmental score** and the **fundamental environmental value**, when the company is accurately rated or already overrated.

Greenwashing

Company i is *greenwashing* at time t if:

- (i) it is not underrated, that is, $E_t^i \geq V_t^i$,
- (ii) its environmental communication is positive, $c_t^i > 0$,
- (iii) it communicates more than it abates, $c_t^i > r_t^i$.

When the company is greenwashing, its *greenwashing effort* is defined as

$$c_t^i - r_t^i.$$

Investor's program

Notations: all variables are $\in \mathbb{R}^n$ in this slide.

$$\sup_{\omega \in \mathbb{A}^\omega} \mathbb{E} \left[\int_0^\infty e^{-rt} \left\{ \underbrace{\omega'_t dS_t - \frac{\gamma}{2} \langle \omega' dS \rangle_t}_{\text{Mean-variance criterion}} + \underbrace{\omega'_t (\beta E_t - \alpha M_t) dt}_{\text{Non-pecuniary preferences}} \right\} \right]$$

Mean-variance criterion (Standard, e.g., Bouchard et al., 2018)

Non-pecuniary preferences:

- Pro-environmental preferences, βE_t (e.g., Pástor et al., 2021; Zerbib, 2022)
- Penalty on revealed misrating, $-\alpha M_t$

Company i 's program

Notations: the exponent i indicates the i -th component of a vector.

Objective: Trade-off between reducing its **cost of capital** μ^i and the **quadratic costs** of environmental efforts

$$\inf_{(r^i, c^i) \in \mathbb{A}} \mathbb{E} \left[\int_0^{\infty} e^{-\delta t} \left(\mu_t^i + \frac{\kappa_r^i}{2} (r_t^i)^2 + \frac{\kappa_c^i}{2} (c_t^i)^2 \right) dt \right],$$

- μ_t^i : expected returns of company i determined at equilibrium
- $\frac{\kappa_r^i}{2} (r_t^i)^2$: quadratic costs of abatement effort, r_t^i
- $\frac{\kappa_c^i}{2} (c_t^i)^2$: quadratic costs of communication effort, c_t^i

→ Use of expected returns rather than prices because: (i) critical financial variable affected by companies' investments in sustainable projects (Pástor et al., 2021; Zerbib, 2022; Angelis et al., 2022), (ii) similar equivalent formulation (consistent with the literature, McConnell and Sandberg, 1975 and Nantell and Carlson, 1975); (iii) allows for closed-form formulas; (iv) gaussian prices yield expected returns in dollar terms homogeneous with costs.

Structure of the game

Stackelberg equilibrium in the game between companies (leaders) and the investor (follower):

- 1. The investor determines her optimal asset allocation and companies' expected returns** (as the market clears), given her expectation on companies' stock prices, environmental scores, and score for environmental misrating.
- 2. Companies choose their optimal communication and abatement policies** given their expected returns/costs of capital, abatement costs, and communication costs.

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Optimal portfolio and equilibrium expected returns

Proposition

The optimal asset allocation of the investor is the pointwise solution

$$\omega_t^* = \frac{1}{\gamma} \Sigma^{-1} (\mu_t + \beta E_t - \alpha M_t),$$

and the equilibrium expected return is

$$\mu_t = \gamma \Sigma \mathbf{1}_n - \beta E_t + \alpha M_t.$$

βE_t : Green premium on expected returns (Pástor et al., 2021; Zerbib, 2022).

αM_t : Additional correction for greenwashing companies.

Companies' program with explicit objective

Knowing equilibrium expected returns, companies' program becomes:

$$\inf_{(r^i, c^i) \in \mathbb{A}} \mathbb{E} \left[\int_0^\infty e^{-\delta t} \left(\gamma \Sigma \mathbf{1}_n - \beta E_t^i + \alpha M_t^i + \frac{\kappa_r^i}{2} (r_t^i)^2 + \frac{\kappa_c^i}{2} (c_t^i)^2 \right) dt \right].$$

Under the following constraints:

$$\begin{cases} dE_t^i = a(V_t^i - E_t^i)dt + b(V_{t-}^i - E_{t-}^i)dN_t^i + c_t^i dt + z dW_t^i, & E_0^i = q^i, \\ dV_t^i = r_t^i dt, & V_0^i = p^i, \\ dM_t^i = -\rho M_t^i dt + b^2(V_{t-}^i - E_{t-}^i)^2 dN_t^i, & M_0^i = u^i, \end{cases}$$

$$\mathbb{A} := \left\{ (c, r) \in \mathbb{R}^2, \mathbb{F}\text{-prog. meas.} : \mathbb{E} \left[\int_0^\infty e^{-\delta' \wedge \delta t} (|c_t|^2 + |r_t|^2) dt \right] < \infty \right\}$$

\Rightarrow Each company looks for r^i and c^i that maximize its environmental score, E^i , controlling for its misrating score, M^i , and costs of environmental action (abatement and communication), $\frac{\kappa_r^i}{2} (r_t^i)^2 + \frac{\kappa_c^i}{2} (c_t^i)^2$.

Optimal strategies

Proposition (Optimal strategies)

The optimal environmental communication effort, $c^{i,*}$, and abatement effort, $r^{i,*}$, of company i are as follows:

$$c_t^{i,*} = \frac{1}{\kappa_c^i} \left(B^i - A^i (E_t^{i,*} - V_t^{i,*}) \right),$$

$$r_t^{i,*} = \frac{1}{\kappa_r^i} \left(\frac{\beta}{\delta} - B^i + A^i (E_t^{i,*} - V_t^{i,*}) \right),$$

where

$$B^i = \frac{\beta(1 + \frac{A^i}{\delta \kappa_r^i})}{\delta + a + b\lambda^i + \frac{2A^i}{\bar{\kappa}_r^i}}, \quad A^i = \frac{\bar{\kappa}_r^i}{4} R^i \left(\sqrt{1 + \frac{16}{\bar{\kappa}_r^i} \frac{T^i}{(R^i)^2}} - 1 \right)$$

$$T^i = \frac{\lambda^i b^2 \alpha}{\delta + \rho}, \quad R^i = \delta + 2a + \lambda^i (1 - (1 - b)^2), \quad \bar{\kappa}_r^i = \frac{2}{\frac{1}{\kappa_r^i} + \frac{1}{\kappa_c^i}}$$

with $E^{i,*}$, $V^{i,*}$ state variables when the optimal strategies $c^{i,*}$, $r^{i,*}$ are employed, $A^i, B^i \geq 0$ and $\frac{\beta}{\delta} - B^i \geq 0$.

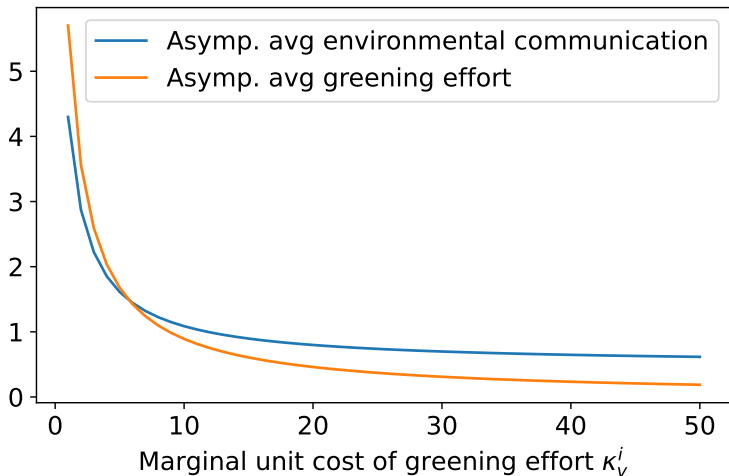
Optimal strategies

Emissions abatement and environmental communication of company i jointly serve the purpose of **increasing its environmental score without decoupling it too much from its fundamental environmental value.**

Summary of the main forces at play:

- c^i and r^i decrease with their marginal unit cost of abatement, κ_c^i and κ_r^i
- “Incentive force”: $B^i > 0$ for c^i and $(\frac{\beta}{\delta} - B^i) > 0$ for r^i , which both increase with pro-environmental preferences, β
- “Corrective force”: A^i , which aims at limiting the level of misrating in response to the investor's penalty on misrating with intensity α
- Interaction effect that aims at keeping both strategies sufficiently close to each other to limit the penalty

Illustration of the interaction effect on $\lim_{t \rightarrow \infty} \mathbb{E}[c_t^{i,*}]$ and $\lim_{t \rightarrow \infty} \mathbb{E}[r_t^{i,*}]$



Marginal benefit of a strategy

Define the functional $J(c, r)$ as the expected discounted integral of the cost of capital:

$$J(c, r) := \mathbb{E} \left[\int_0^{\infty} e^{-\delta t} \{ -\gamma \Sigma \mathbf{1}_n + \beta E_t^{c,r} - \alpha M_t^{c,r} \} dt \right],$$

and its **Fréchet derivatives** in c and r be written as: $(\Pi_t^c)_{t \geq 0}$, $(\Pi_t^r)_{t \geq 0}$.

Definition (Marginal benefit)

The marginal benefits of increasing communication or abatement at a given time t are defined as Π_t^c and Π_t^r respectively.

⇔ Impact on the integrated discounted cost of capital of increasing communication or abatement over an infinitesimal time interval.

Marginal benefit of a strategy

Proposition

Let an admissible strategy (c^i, r^i) , and the corresponding state variables (E^i, V^i) .

Marginal benefit of increasing **communication** at time t :

$$\Pi_t^{c^i, i} = \frac{\beta}{\delta + a + b\lambda^i} - 2T^i \mathbb{E} \left[\int_t^\infty e^{-(\delta+a)(s-t)} (1-b)^{N_s - N_t} (E_s^i - V_s^i) ds \middle| \mathcal{F}_t \right].$$

Marginal benefit of increasing **abatement** at time t :

$$\Pi_t^{r^i, i} = \frac{\beta}{\delta} - \frac{\beta}{\delta + a + b\lambda^i} + 2T^i \mathbb{E} \left[\int_t^\infty e^{-(\delta+a)(s-t)} (1-b)^{N_s - N_t} (E_s^i - V_s^i) ds \middle| \mathcal{F}_t \right].$$

Moreover, **at optimum**, the strategies verify:

$$\Pi_t^{c^{i,*}, i} = \kappa_c^i c_t^{i,*}, \quad \Pi_t^{r^{i,*}, i} = \kappa_r^i r_t^{i,*}.$$

Constant part: Impact of a rise in communication and abatement on the integrated discounted cost of capital through an increase in the environmental score (increases w/ β).

Stochastic part: Impact of a rise in communication and abatement on the integrated discounted cost of capital as a function of the misrating and its penalty.

Optimal greenwashing effort when $\beta > 0$, $\alpha > 0$

Proposition (Greenwashing effort)

If the following condition (*) is satisfied,

← Is greenwashing relevant?

$$\frac{\kappa_r^i}{\kappa_c^i} > \frac{a + b\lambda^i}{\delta}, \quad (*)$$

company i greenwashes if, and only if,

← Is greenwashing beneficial?

$$0 \leq E_t^{i,*} - V_t^{i,*} < \frac{1}{\frac{2}{\bar{\kappa}^i} A^i} G_{max}^i, \quad G_{max}^i = \frac{2}{\bar{\kappa}^i} B^i - \frac{\beta}{\delta \kappa_r^i}.$$

When it greenwashes, its greenwashing effort is as follows:

$$c_t^{i,*} - r_t^{i,*} = G_{max}^i - \frac{2}{\bar{\kappa}^i} A^i (E_t^{i,*} - V_t^{i,*})$$

When condition (*) is not satisfied, company i never greenwashes.

NB: $a + b\lambda^i \equiv$ Revelation intensity (inverse: degree of **information asymmetry**).

⇒ Companies greenwash to maintain their environmental score at a certain level above their environmental value = maximal greenwashing effort discounted by the company's effort to reduce its overrating, $\frac{2}{\bar{\kappa}^i} A^i$.

Greenwashing impact

Definition (Greenwashing impact)

The impact of company i 's greenwashing strategy is defined as:

$$\lim_{t \rightarrow \infty} \mathbb{E}[E_t^{i,*} - V_t^{i,*}].$$

Proposition (Greenwashing impact)

When condition (*) is satisfied, the impact of company i 's greenwashing strategy is as follows:

$$\lim_{t \rightarrow \infty} \mathbb{E}[E_t^{i,*} - V_t^{i,*}] = \frac{1}{\frac{2}{R^i} A^i + a + b\lambda^i} G_{max}^i,$$

where the convergence takes place with an exponential rate.

⇒ Greenwashing impact = overrating target, $\frac{1}{\frac{2}{R^i} A^i} G_{max}^i$, further discounted by the revelation intensity, $a + b\lambda^i$, over the period

Impact of investor's preferences and penalty

β Sensitivity of pro-environmental preferences of the investor

α Investor's penalty on revealed misrating

Proposition (Investor's impact on greenwashing)

When condition () is satisfied, the maximal greenwashing effort, G_{max}^i , increases linearly in β and decreases in a convex way in α .*

Proposition (Investor's impact on abatement)

The constant part of the optimal abatement effort, $\frac{1}{\kappa_r^i} \left(\frac{\beta}{\delta} - B^i \right)$, increases linearly in β , and, when condition () is satisfied, increases in a concave way in α .*

⇒ Adds to the impact investing literature (Landier and Lovo, 2023; Green and Roth, 2024; Pástor et al., 2022; De Angelis et al., 2023; Oehmke and Opp, 2024).

The impact of investment decisions on greenwashing and abatement

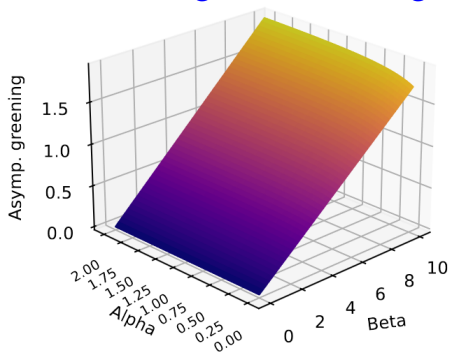
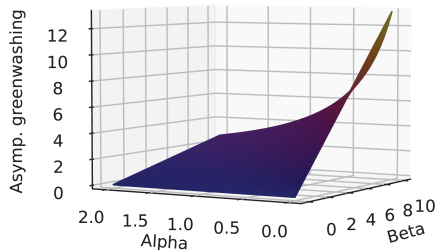


Figure: Average greenwashing and abatement as a function of β and α . Asymptotic expected optimal greenwashing ($\lim_{t \rightarrow \infty} \mathbb{E}[c_t^* - r_t^*]$; left) and abatement ($\lim_{t \rightarrow \infty} \mathbb{E}[r_t^*]$; right) as a function of the pro-environmental sensitivity, β , and the misrating penalty, α .

- ⇒
- Greenwashing and abatement efforts increase linearly with green preferences β .
 - Penalty α strongly deters greenwashing, and encourages abatement.

Greenwashing and transparency parameters

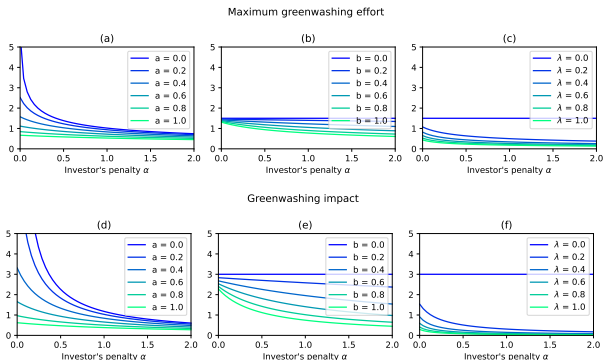


Figure: Greenwashing and penalty α for various transparency parameters. The maximum greenwashing effort, G_{max}^i , and greenwashing impact, $\lim_{t \rightarrow \infty} \mathbb{E}[E_t^{i,*} - V_t^{i,*}]$, as a function of the investor's penalty, α , for different values of transparency parameters a , b , λ^i .

When investors penalize misrating ($\alpha > 0$):

- a plays as a *substitute* for the penalty α
- λ^i and b have *complementary* effects to the penalty α .

Greenwashing and technological change

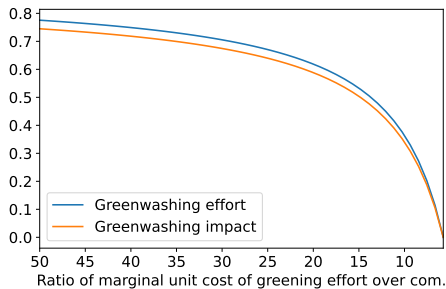


Figure: Greenwashing and technological change. Maximum greenwashing effort, G_{max}^i , and impact, $\lim_{t \rightarrow \infty} \mathbb{E}[E_t^{i,*} - V_t^{i,*}]$, in function of the ratio of marginal unit costs of abatement and communication κ_r^i / κ_c^i . Consistently with Proposition 4.4, greenwashing is zero when the threshold represented by condition (*) is hit.

⇒ Curbing greenwashing through green technological change would require a sustained and pronounced R&D effort to bring down κ_r^i before being effective on greenwashing effort and impact. (With our calibration the ON-OFF condition is shut-down when the ratio equals 5.)

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Empirical analysis

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However, we build a proxy for environmental communication effort, \hat{c}_t^i , and:

1. analyze its strength;
2. test the dynamics of the model:

$$c_t^{i,*} = \frac{1}{\kappa_c^i} \left(B^i - A^i (E_t^{i,*} - V_t^{i,*}) \right)$$

Empirical analysis

Challenge: No robust, exhaustive, and dynamic data on companies' emission abatement. \Rightarrow Unreliable test for greenwashing

However, we build a proxy for environmental communication effort, \hat{c}_t^i , and:

1. analyze its strength;
2. test the dynamics of the model:

$$c_t^{i,*} = \frac{1}{\kappa_c^i} \left(B^i - A^i (E_t^{i,*} - V_t^{i,*}) \right)$$

Monthly data from Covalence:

- an environmental reputation score, $Rep \in [0, 100]$;
- an environmental controversy score, $Con \in [0, 100]$;
- an environmental performance score, $E \in [0, 100]$.

Sample: 3,769 global companies between December 2015 and December 2022: 145,508 firm \times month observations.

Empirical Method

We build a **two-step method**:

- **Step 1:** Build a proxy for the environmental communication effort, out of *Rep* and *Con*
⇒ Analyze \hat{C}_t^i
- **Step 2:** Test the dynamics of environmental communication effort
⇒ Test the equilibrium equation based on \hat{C}_t^i

Method: Step 1 (Proxy for environmental comm. effort)

Step 1: Proxy for the environmental communication effort

Idea: **Proxy** = **orthogonal** component of the **environmental reputation** score to the **environmental controversy** score.

Method: Step 1 (Proxy for environmental comm. effort)

Step 1: Proxy for the environmental communication effort

Idea: **Proxy** = **orthogonal** component of the **environmental reputation** score to the **environmental controversy** score.

- Estimated specification, with **instrumentation** to address the *simultaneity bias*:

$$Rep_t^i = \alpha_1^i + \beta_1 Con_t^{i,*} + \varepsilon_{1,t}^i,$$

where $Con_t^{i,*}$ is the prediction of the following regression:

$$Con_t^i = \alpha_2^i + \beta_2 Con_{t-1}^i + \varepsilon_{2,t}^i.$$

- **Resulting proxy** for the flow of monthly communication:

$$\hat{C}_t^i \equiv (\hat{\alpha}_1^i + \hat{\varepsilon}_{1,t}^i) - (\hat{\alpha}_1^i + \hat{\varepsilon}_{1,t-1}^i) = \hat{\varepsilon}_{1,t}^i - \hat{\varepsilon}_{1,t-1}^i$$

Method: Step 1 (Proxy for environmental comm. effort)

Comments on the step-1 regression:

- Con_t^i is **relevant instrument**: the R^2 of the regression of Con_t^i on Con_{t-1}^i is 76.4%, and the correlation between both variables is 81.3%;

- **Weak exogeneity**:

$\forall i \in \{1, \dots, n\}, \forall (t', t) \in \{1, \dots, T\}^2, t' \geq t, \mathbb{E}(\varepsilon_{1,t'}^i, Con_t^{i,*}) = 0$, because

$\forall i \in \{1, \dots, n\}, \forall t \in \{1, \dots, T\}, \forall j \in \{1, \dots, t-1\}, \mathbb{E}(\varepsilon_{1,t}^i, Con_{t-j}^i) = 0$.

Intuition: The shocks to environmental reputation scores at the end of month t , $\varepsilon_{1,t}^i$, are uncorrelated with controversies that took place during month $t-j$, with $j \in \{1, \dots, t-1\}$.

Lemma

The bias of the Within estimate under weak exogeneity tends towards zero at a rate faster than or equal to $1/T$.

We perform the estimations using 84 and 120 dates.

Method: Step 2 (Dynamics of env. comm. effort)

Recall, we want to test:

$$c_t^{j,*} = \frac{1}{k_c^j} \left(B^j - A^j (E_t^{j,*} - V_t^{j,*}) \right)$$

Challenge: V_t^j is **unobservable** and probably correlated with E_t^j .

Method: Step 2 (Dynamics of env. comm. effort)

Recall, we want to test:

$$c_t^{j,*} = \frac{1}{\kappa_c^j} \left(B^j - A^j (E_t^{j,*} - V_t^{j,*}) \right)$$

Challenge: V_t^i is **unobservable** and probably correlated with E_t^i .

Idea: we can test the **time derivative** (first diff.) of c_t^i by making the reasonable assumption that the V_t^i is **highly inert from one month to the next**. Hence,

$$\frac{1}{\kappa_c} A^i \Delta V_t^i = \eta_1^i + \eta_{2,t}^i \quad (\eta_{2,t}^i \text{ error term}),$$

and to address simultaneity issues, we estimate:

$$\Delta \hat{c}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta E_t^{i,*} + \varepsilon_{3,t}^i,$$

where $\Delta E_t^{i,*}$ is the prediction of the following regression:

$$\Delta E_t^i = \alpha_4^i + \beta_4 E_{t-2}^i + \varepsilon_{4,t}^i.$$

Method: Step 2 (Dynamics of env. comm. effort)

Comments on the step-2 regression:

- E_{t-2}^i is **relevant and strong instrument**.

- **Weak exogeneity:**

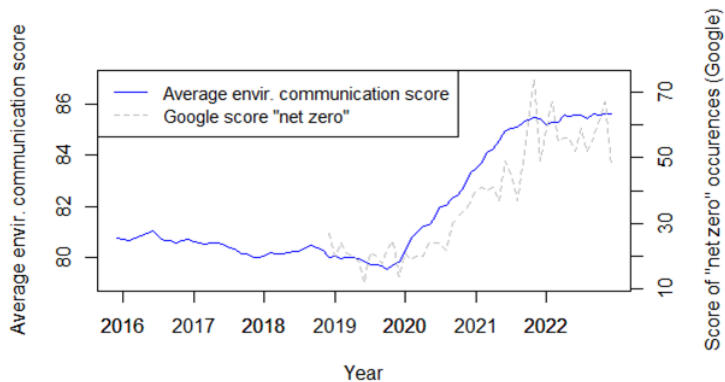
$\forall i \in \{1, \dots, n\}, \forall (t', t) \in \{1, \dots, T\}^2, t' \geq t, \mathbb{E}(\varepsilon_{3,t'}^i \Delta E_t^{i,*}) = 0$, because

$\forall i \in \{1, \dots, n\}, \forall t \in \{1, \dots, T\}, \forall j \in \{2, \dots, t-1\}, \mathbb{E}(\varepsilon_{3,t}^i E_{t-j}^i) = 0$.

Intuition: The shocks to the change in communication flow between month t and month $t+1$, $\varepsilon_{3,t}^i$, are uncorrelated with the environmental scores set at the end of month $t-j$, with $j \in \{2, \dots, t-1\}$.

- Same comment as above regarding the **convergence** of the Within estimator under weak exogeneity.

Estimation: Step 1 (Environmental communication)

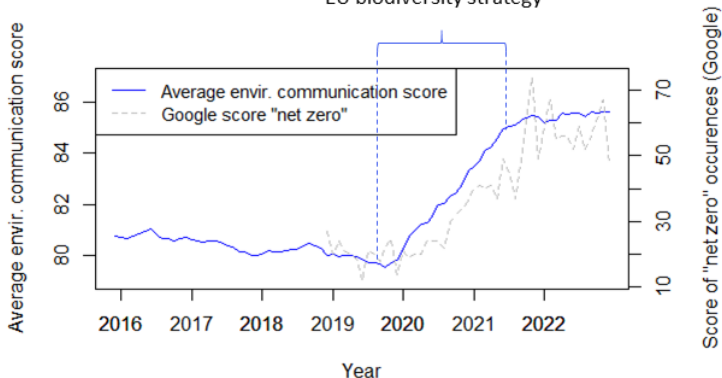


Estimation: Step 1 (Environmental communication)

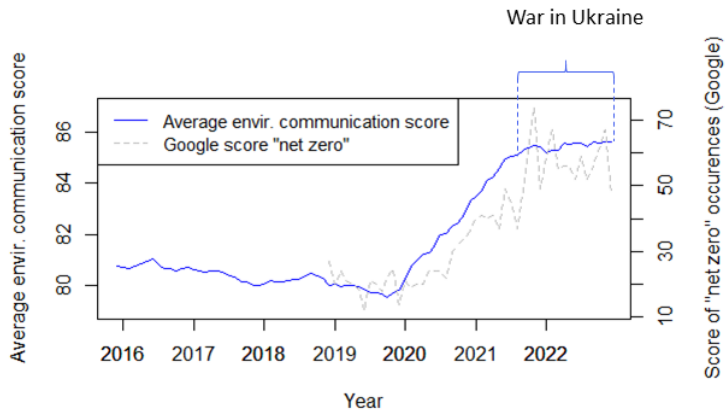
Key environmental regulations worldwide.

E.g., EU:

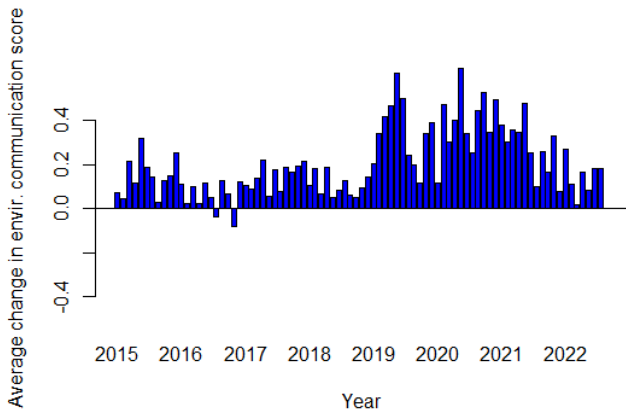
- EU Green Deal, regulations on binding annual emission reductions,
- circular economy,
- sustainable finance,
- EU biodiversity strategy



Estimation: Step 1 (Environmental communication)



Estimation: Step 1 (Environmental comm effort, \hat{c}_t^i)



⇒ **98.8% of the average monthly environmental communication over the period is positive.**

$$\text{Estimation: Step 2 } (\Delta \hat{c}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta E_t^{i,*} + \varepsilon_{3,t}^i)$$

Dependent variable: $\Delta \hat{c}_t^i$					
Top brownest companies:					
	10%	20%	30%	40%	50%
$\Delta E_t^{i,*}$	-0.071 (0.051)	-0.164** (0.065)	-0.244*** (0.073)	-0.221*** (0.067)	-0.271*** (0.060)
Firm FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
Observations	18,760	30,711	44,116	56,785	68,276
R ²	0.005	0.006	0.008	0.010	0.013
Adjusted R ²	-0.061	-0.049	-0.041	-0.035	-0.029
F Statistic	0.985	3.525*	5.460**	3.608*	4.949**
Dependent variable: $\Delta \hat{c}_t^i$					
Top brownest companies:					
	60%	70%	80%	90%	Whole sample
$\Delta E_t^{i,*}$	-0.237*** (0.053)	-0.176*** (0.049)	-0.188*** (0.046)	-0.158*** (0.040)	-0.119*** (0.033)
Firm FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
Observations	83,309	97,324	110,206	123,864	145,508
R ²	0.015	0.016	0.017	0.017	0.017
Adjusted R ²	-0.023	-0.019	-0.015	-0.012	-0.008
F Statistic	3.476*	1.756	1.875	1.195	0.661

Note:

*p<0.1; **p<0.05; ***p<0.01

$$\text{Estimation: Step 2 } (\Delta \hat{c}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta E_t^{i,*} + \varepsilon_{3,t}^i)$$

Dependent variable: $\Delta \hat{c}_t^i$					
Top greenest companies:					
	10%	20%	30%	40%	50%
$\Delta E_t^{i,*}$	-0.255*** (0.079)	-0.342*** (0.069)	-0.446*** (0.072)	-0.405*** (0.061)	-0.415*** (0.057)
Firm FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
Observations	21,644	35,302	48,184	62,199	77,232
R ²	0.018	0.019	0.021	0.020	0.020
Adjusted R ²	-0.018	-0.013	-0.010	-0.010	-0.009
F Statistic	4.284**	8.542***	14.584***	11.377***	10.606***
Dependent variable: $\Delta \hat{c}_t^i$					
Top greenest companies:					
	60%	70%	80%	90%	Whole sample
$\Delta E_t^{i,*}$	-0.404*** (0.052)	-0.380*** (0.054)	-0.294*** (0.052)	-0.237*** (0.044)	-0.119*** (0.033)
Firm FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
Observations	88,723	101,392	114,797	126,748	145,508
R ²	0.022	0.022	0.022	0.021	0.017
Adjusted R ²	-0.007	-0.006	-0.006	-0.006	-0.008
F Statistic	8.727***	6.709***	3.513*	2.169	0.661

Note:

*p<0.1; **p<0.05; ***p<0.01

Testing the equation of optimal communication

⇒ Companies, **especially the greenest ones**, use **environmental communication in a counter-cyclical way** with respect to the evolution of their environmental score, in line with the results of the model.

The results are **robust** to:

- **Controlling for systematic risks and returns.** →
- Repeating the estimation starting at **different dates**: December 2012, December 2017, December 2019, and December 2021. →
- Using 3 environmental **subscores** related to (i) the environmental impacts of the products sold, (ii) the resources used, and (iii) the emissions, effluents, and waste. →

What about greenwashing?

Conclusions about environmental communication:

1. Companies have implemented a **quasi-structural positive envir. com.** policy
2. **Counter-cyclical dynamic of the envir. com.**, as highlighted by the model

Three possible **interpretations**:

1. Companies are **structurally underrated**.
→ But no evidence of underrating; in addition evidence that rating agencies tend to be biased in favor of borrowers (Manso, 2013)
2. Companies use communication to support their **continuous abatement effort**.
→ But monthly communication is very likely to be more volatile than environmental value.
3. Companies **greenwash** at least part of the time.
→ Supported by the low MUC of communication and the asymmetry of information (Barbalau and Zeni, 2023).

⇒ The **greenwashing** option, at least part of the time, is the most likely.

Table of Contents

- 1 Empirical motivation
- 2 A dynamic equilibrium model with corporate greenwashing
- 3 Optimal greenwashing and investor's impact
- 4 Empirical evidence
- 5 Introducing interaction between companies**

The environmental score which matters is relative

Why?

- Best-in-class investment strategies.
- Rescaling of ESG scores.

Investor's program accounting for the companies' universe:

$$\sup_{\omega \in \mathbb{A}^\omega} \mathbb{E} \left[\int_0^\infty e^{-rt} \left\{ \underbrace{\omega'_t dS_t - \frac{\gamma}{2} \langle \omega' dS \rangle_t}_{\text{Mean-variance criterion}} + \underbrace{\omega'_t \left(\beta \frac{E_t}{h(\frac{1}{n} \sum_i E_t^i)} - \alpha M_t \right) dt}_{\text{Non-pecuniary preferences}} \right\} \right],$$

h a regular function approximating identity on \mathbb{R}_+ .

Equilibrium expected returns with this new program:

$$\mu_t = \gamma \Sigma \mathbf{1}_\infty - \beta \frac{E_t}{h(\frac{1}{n} \sum_i E_t^i)} + \alpha M_t.$$

The Greenwashing n -player game

Company i 's program is now interacting with other companies' programs:

$$\inf_{(r^i, c^i) \in \mathbb{A}} \mathbb{E} \left[\int_0^\infty e^{-\delta t} \left(\gamma \Sigma \mathbf{1}_\infty - \beta \frac{E_t^i}{h(\frac{1}{n} \sum_i E_t^i)} + \alpha M_t^i + \frac{\kappa_r}{2} (r_t^i)^2 + \frac{\kappa_c}{2} (c_t^i)^2 \right) dt \right].$$

⇒ ISSUE: No more linear quadratic objective.

⇒ To solve this game, formulation at the **mean-field limit** (i.e., when $n \rightarrow \infty$).

⇒ A generic company does not have any impact on the average environmental score. Hence, linear quadratic program with

$m : t \mapsto \lim_{n \rightarrow \infty} \frac{1}{n} \sum_i E_t^i$ deterministic.

Additional assumptions:

- Atomic and identical companies.
- Idiosyncratic and identically distributed noises $(W^i, N^i)_i$.

Companies' mean field program

The program of the representative company becomes, with finite horizon:

$$\inf_{(r,c) \in \mathbb{A}} \mathbb{E} \left[\int_0^T e^{-\delta s} \left(\gamma \Sigma \mathbf{1}_\infty - \beta \frac{E_t}{h(m_t)} + \alpha M_t + \frac{\kappa_r}{2} (r_t)^2 + \frac{\kappa_c}{2} (c_t)^2 \right) dt \right].$$

Definition (Mean field equilibrium)

Let $J(r, c, m)$ be the objective functional of the firm. Then, (r^*, c^*, m^*) is a mean field equilibrium if, and only if,

- (i) $\forall (r, c) \in \mathbb{A}_T, \quad J(r^*, c^*, m^*) \leq J(r, c, m^*),$
- (ii) $\forall t \in [0, T], \quad m_t^* = \mathbb{E}[E_t^*].$

Optimal strategy for a given population flow

Proposition (Optimal strategies)

For a given population flow m , the optimal environmental communication effort, c^* , and abatement effort, r^* , of the representative company are as follows:

$$c_t^* = \frac{1}{\kappa_c} (B(t) - A(t)(E_t^* - V_t^*)),$$

$$r_t^* = \frac{1}{\kappa_r} \left(\int_t^T \frac{\beta}{h(m_u)} du - B(t) + A(t)(E_t^* - V_t^*) \right),$$

where

$$B(t) = \beta \int_t^T e^{\int_t^s (\frac{2}{\kappa} A(u) - a - \lambda b) du} \left(\frac{1}{h(m_s)} - \frac{A(s)}{\kappa_r} \int_s^T \frac{1}{h(m_u)} du \right) ds$$

and A is the unique solution, negative, of the Riccati equation

$$\dot{A}(t) + \frac{2}{\kappa} A(t)^2 - (2a + \lambda(1 - (1 - b)^2)) A(t) + 2\lambda b^2 \left(\frac{\alpha}{\rho} e^{-\rho(T-t)} - \frac{\alpha}{\rho} \right) = 0, \quad A(T) = 0,$$

and with E^* , V^* state variables when the optimal strategies c^* , r^* are employed.

Existence and uniqueness of a Nash equilibrium

Proposition (Existence and uniqueness of the NE)

Assume that the function h is positive, increasing, and superior to 1. Then, there exists a unique mean field equilibrium.

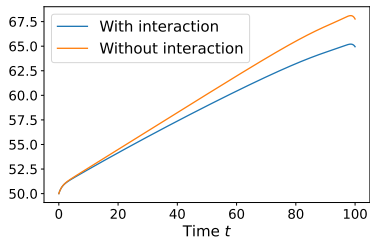
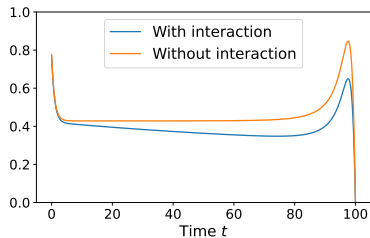
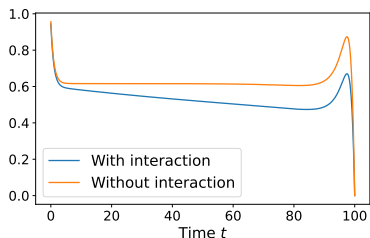
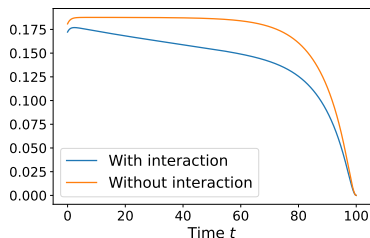
Proof.

1. Show that there exists a map whose fixed points characterize the set of MFE.
2. Existence: Schauder's fixed point theorem.
3. Uniqueness: Lasry-Lions monotonicity condition.

Numerical approximation of the equilibrium: Fictitious play algorithm. With Ψ the fixed point map

$$m_{k+1} = \frac{1}{k+1} \Psi(m_k) + \frac{k}{k+1} m_k.$$

Results: Average abatement, communication, and greenwashing efforts; average environmental score



Conclusion

- **Investors' pro-environmental preferences** incentivize companies to **greenwash**
 - ▶ To the detriment of further abatement
- But **investors can curb greenwashing practices** by penalizing misrating revealed by controversies
 - ▶ This, in turn, spurs abatement
- **Policymakers** can also **curb greenwashing** and **increase abatement**:
 - (i) regulations strengthening **transparency**
 - (ii) support for environmental **technological innovation**
- **Empirical results** suggest that companies tend to **greenwash significantly**.
- These results seem qualitatively **robust** to the introduction of an **interaction** between companies.

Thank you!



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Companies' program in terms of asset prices

Company i 's program is equivalent to the following:

$$\sup_{(r^i, c^i) \in \mathbb{A}} \mathbb{E} \left[\int_0^\infty e^{-\delta t} \left(\delta(S_0^i - S_t^i) - \frac{\kappa_r^i}{2} (r_t^i)^2 - \frac{\kappa_c^i}{2} (c_t^i)^2 \right) dt \right],$$

with S_0^i the initial price of the asset issued by company i .

Equilibrium expected returns: Sketch of the proof

Definition (Equilibrium expected returns)

μ so that:

- the investor implements her optimal investing strategy ω^* ,
- market clears: $\forall i, \forall t, \omega_t^{*,i} = 1$.

Proof.

- Define the candidate optimal strategy $\omega_t^* := \frac{1}{\gamma} \Sigma^{-1} (\mu_t + \beta E_t - \alpha M_t)$.
- The investor's program can be rewritten as

$$\sup_{\omega \in \mathbb{A}^\omega} \mathbb{E} \left[\int_0^\infty e^{-\delta t} \left\{ -\frac{\gamma}{2} (\omega_t - \omega_t^*)' \Sigma (\omega_t - \omega_t^*) + \frac{\gamma}{2} \omega_t^{*'} \Sigma \omega_t^* \right\} dt \right].$$

\Rightarrow The optimal portfolio choice of the investor is thus the pointwise solution ω_t^* .

- In addition, writing $\mathbf{1}_n$ a vector of ones of size n , market clearing condition writes:
 $\forall t, \omega_t^* = \mathbf{1}_n$.
- Equilibrium expected returns are therefore $\mu_t = \gamma \Sigma \mathbf{1}_n - \beta E_t + \alpha M_t$.

Sketch of the proof

1. Show that, at optimum, optimal strategies verify the following: $\kappa_c^i c_t^{i,*} + \kappa_r^i r_t^{i,*} = \frac{\beta}{\delta}$.
2. Reduce the dimension of the problem by a change of variable:
 - ▶ State variables: $(E, V, M) \Rightarrow (X, M)$, $X := E - V$ (overrating)
 - ▶ Controls: $(c, r) \Rightarrow \xi$, $\xi := c - r$ (greenwashing effort)
 - ▶ Equivalent program:

$$\sup_{\substack{\xi=c-r, \\ (r,c) \in \mathbb{A}}} \mathbb{E} \left[\int_0^\infty e^{-\delta t} \left(\beta X_t^x - \alpha M_t^u - \frac{\bar{\kappa}}{4} \left(\xi_t + \frac{\beta}{\delta \kappa_r} \right)^2 \right) dt \right].$$

3. Solve the equivalent program with **one-dimensional** control variable. HJB equation:

$$\max_{\xi \in \mathbb{R}} \left\{ \beta x - \alpha u - \frac{\bar{\kappa}}{4} \left(\xi + \frac{\beta}{\delta \kappa_r} \right)^2 - \delta v + \frac{\partial v}{\partial x} (-ax + \xi) - \frac{\partial v}{\partial u} \rho u + \frac{z^2}{2} \frac{\partial^2 v}{\partial x^2} + \lambda \left[v(x(1-b), u + b^2 x^2) - v(x, u) \right] \right\} = 0.$$

4. Deduce optimal strategies in the optimal problem using equality stated in 1.

Robustness: Controls

Dependent variable: Δc_t^g					
Top greenest companies:					
	10%	20%	30%	40%	50%
$\Delta E_t^{c,*}$	-0.205 (0.182)	-0.380** (0.178)	-0.261* (0.142)	-0.243** (0.096)	-0.280*** (0.093)
R_{t-1}^g	-0.335 (0.287)	-0.222 (0.245)	-0.002 (0.217)	0.348 (0.241)	0.480** (0.232)
$\beta_{t-1}^{CAPM,i}$	0.005 (0.015)	0.008 (0.014)	-0.013 (0.027)	0.008 (0.013)	-0.009 (0.014)
Firm FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	8,084	12,272	16,003	19,503	23,219
R ²	0.016	0.021	0.023	0.022	0.020
Adjusted R ²	-0.023	-0.012	-0.008	-0.009	-0.009
F Statistic	1.504	3.582	1.748	3.120	5.449
Dependent variable: Δc_t^g					
Top greenest companies:					
	60%	70%	80%	90%	Whole sample
$\Delta E_t^{c,*}$	-0.385*** (0.093)	-0.284*** (0.086)	-0.251*** (0.093)	-0.193*** (0.067)	-0.083* (0.050)
R_{t-1}^g	0.375* (0.220)	0.185 (0.170)	0.316* (0.171)	0.255* (0.153)	0.252** (0.124)
$\beta_{t-1}^{CAPM,i}$	0.005 (0.011)	0.008 (0.011)	-0.011 (0.012)	-0.0002 (0.010)	0.010 (0.007)
Firm FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	25,745	28,779	32,062	35,208	41,252
R ²	0.023	0.022	0.023	0.022	0.016
Adjusted R ²	-0.007	-0.007	-0.006	-0.006	-0.012
F Statistic	5.711	2.722	4.029	2.754	3.014

Note:

*p<0.1; **p<0.05; ***p<0.01

Robustness: Period

Dependent variable: $\Delta \tilde{c}_t^i$				
50% brownest companies				
	Since 2012	Since 2017	Since 2019	Since 2021
$\Delta E_t^{i,*}$	-0.271*** (0.060)	-0.226*** (0.057)	-0.220*** (0.072)	-0.237*** (0.087)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Observations	68,276	57,626	43,107	19,098
R ²	0.013	0.014	0.019	0.022
Adjusted R ²	-0.029	-0.034	-0.042	-0.093
F Statistic	4.949**	3.497*	3.420*	4.817**
Dependent variable: $\Delta \tilde{c}_t^i$				
50% greenest companies				
	Since 2012	Since 2017	Since 2019	Since 2021
$\Delta E_t^{i,*}$	-0.415*** (0.057)	-0.457*** (0.061)	-0.449*** (0.065)	-0.353*** (0.069)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Observations	77,232	64,719	48,000	20,768
R ²	0.020	0.022	0.026	0.029
Adjusted R ²	-0.009	-0.012	-0.020	-0.075
F Statistic	10.606***	13.629***	18.549***	9.557***

Note: *p<0.1; **p<0.05; ***p<0.01

Robustness: Subscores

	Dependent variable: $\Delta \hat{c}_t^i$			Dependent variable: $\Delta \hat{c}_t^i$		
	50% brownest companies			50% greenest companies		
	(1)	(2)	(3)	(1)	(2)	(3)
$\Delta E_t^{Imp,i,*}$	-0.142*** (0.046)			$\Delta E_t^{Imp,i,*}$	-0.269*** (0.042)	
$\Delta E_t^{Res,i,*}$		-0.180*** (0.047)		$\Delta E_t^{Res,i,*}$		-0.252*** (0.038)
$\Delta E_t^{Emi,i,*}$			-0.204*** (0.051)	$\Delta E_t^{Emi,i,*}$		-0.225*** (0.036)
Firm FE	Yes	Yes	Yes	Firm FE	Yes	Yes
Time FE	Yes	Yes	Yes	Time FE	Yes	Yes
Observations	68,276	68,276	68,276	Observations	77,232	77,232
R ²	0.006	0.005	0.015	R ²	0.013	0.009
Adjusted R ²	-0.036	-0.037	-0.027	Adjusted R ²	-0.016	-0.020
F Statistic	2.087	3.580*	3.978**	F Statistic	5.953**	8.354***

Note:

*p<0.1; **p<0.05; ***p<0.01

Directional marginal benefits

Let $\epsilon > 0$. For a pair of communication and abatement strategies $c, r \in \mathbb{A}$ and a pair of test functions $\delta c, \delta r \in \mathbb{A}$, let us define the associated pair of modified strategies:

$$c_s^\epsilon := c_s + \epsilon \delta c_s, \quad r_s^\epsilon := r_s + \epsilon \delta r_s.$$

Define the functional $J(c, r)$ as the expected discounted integral of the cost of capital:

$$J(c, r) := \mathbb{E} \left[\int_0^\infty e^{-\delta t} \{ -\gamma \Sigma \mathbf{1}_n + \beta E_t^{c,r} - \alpha M_t^{c,r} \} dt \right],$$

Then, the expected marginal benefits of communication and abatement along directions δc and δr are defined respectively as the directional (Gateaux) derivatives of J in these two directions:

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (J(c + \epsilon \delta c, r) - J(c, r)), \quad \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (J(c, r + \epsilon \delta r) - J(c, r)).$$

Marginal benefits of emissions reduction and communication

The directional marginal benefits (Gâteaux derivatives) are linear, and can be expressed through Frechet derivatives D_t^c and D_t^r :

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (J(\mathbf{c} + \epsilon \delta \mathbf{c}, r) - J(\mathbf{c}, r)) = \mathbb{E} \left[\int_0^{\infty} e^{-\delta t} D_t^c J(\mathbf{c}, r) \delta \mathbf{c}_t dt \right],$$
$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (J(\mathbf{c}, r + \epsilon \delta r) - J(\mathbf{c}, r)) = \mathbb{E} \left[\int_0^{\infty} e^{-\delta t} D_t^r J(\mathbf{c}, r) \delta r_t dt \right].$$

The derivatives D_t^c and D_t^r shall be called marginal benefits of increasing communication or abatement at a given time t .

Reference calibration

Table: Calibration.

Parameter	Value
a	0.4
b	1
λ	8.5%
κ_C	1
κ_r	50
β	1
α	1
ρ	0.1
δ	0.1
z	0.2