Optimal Dynamic Regulation of Carbon Emissions Market

Workshop on Energy, mathematics, and theoretical challenges
Institut Henri Poincaré, October 2024

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Agenda

Motivations

Market of Carbon Permits

- Carbon permits are tradable certificates that can be purchased on a dedicated market (e.g. EEX in Europe) to compensate for GHG emissions in carbon equivalence terms.
- Each permit offsets one metric ton of CO2, effectively putting a price on pollution. It serves as a market-based alternative to a carbon tax on emissions.
- Debate on what is better: the carbon tax, or a carbon market.

Compliance carbon markets

- Mandatory carbon reduction plans with verified emissions.
- Target: Energy-intensive companies in sectors such as metal, oil and gas, public power and heat, lime and cement, or airlines.
- Mechanism is the Emission Trading Systems (ETS) or cap-and-trade, implemented in EU 27 and other countries (China, Canada)
- European (EU) ETS international level, launched in 2005.

EU ETS Phases

- Launched in 2005, the EU Emissions Trading System covers half of the EU greenhouse gas emissions from over 15,000 heavy energy-using installations, including power stations and industrial plants (oil refineries, metals, cement, lime, glass, ceramics, pulp and paper, acids and chemicals).
- The experimental Phase I ended in 2007. We are now in Phase IV (2021-2030), a resulting fine tuning of the previous Phases.
- In the beginning the allocation was too generous. Later on, part of permits was unnecessary due to the economic recession in 2007–2015. The surplus, from 2005 to 2015, didn't incentive reduction.
- From 2019 the Market Stability Reserve mechanism monitors the total number of circulating allowances.
- If the total number of allowances in circulation (TNAC) falls under 400 milllions, the regulator allocates allowances. If it reaches 833 millions, allocations is stopped.
- The allocation process becomes dynamic.

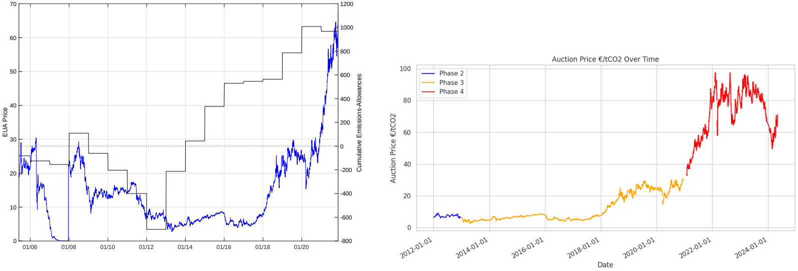

Figure: (Left) EUA price (€/tCO2) and difference between total verified emissions and total allocations (MtCO2) (Right) EUA prices Phase II-IV.

Data

Bob Thomas
Quantitative Economic Analysis Master Student

Data

- Spot and futures market prices via Refinitiv
- Physical installation data available on the European Union Transaction Log (EUTL) website.
- Jan Abrell on EUETS.info to collect the data on the EUTL and the European Commission website.
- Transactions data: The most recent data is displayed on 1 May of the third year after the recording of the information.

Installation FR_207493: Chaufferie d'Appoint Secours CACHAN

Installation Details	
Address	FR_207493: Chaufferie d'Appoint Secours CACHAN 61 avenue du Président Wilson Face n°40 94230 Cachan France
Regulated in	EU Emissions Trading System
Registry	France
Activity	20: Combustion of fuels
Nace	35.30: Steam and air conditioning supply
Official EUTL page	

Operator Holding Accounts

Operator Holding Accounts are the accounts used by the installation to exchange allowances with other participants in the Trading System.

Account	Account Holder	Account Type
IDF Chaufferie d'appoint secours CACHAN	DALKIA	Operator Holding Account

Compliance Graph

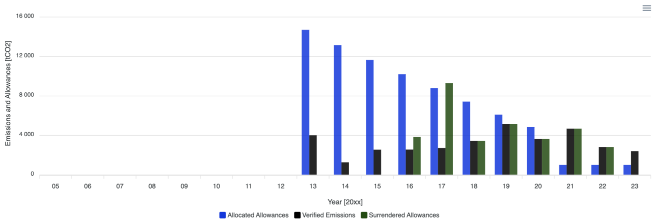

Figure: (Left) An example of data on an installation (Right) Its corresponding annual allowances, verified emissions and surrended emissions.

transactionID	transactionDate	amount	transferringAccountName	acquiringAccountName	transferringAccountIdentifier	acquiringAccountIdentifier	transactionTypeMain_id	transactionTypeSupplementary_id
DK463787	2009-07-08 17:11:55	53548	Fico limited	Unitrade	5590.0	4571.0	10	0
DK463787	2009-07-08 17:11:55	7695	Fico limited	Unitrade	5590.0	4571.0	10	0
DK463787	2009-07-08 17:11:55	7935	Fico limited	Unitrade	5590.0	4571.0	10	0
DK463787	2009-07-08 17:11:55	10000	Fico limited	Unitrade	5590.0	4571.0	10	0
FR110761	2009-07-08 17:15:01	4709	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110761	2009-07-08 17:15:01	1724	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110761	2009-07-08 17:15:01	769	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110761	2009-07-08 17:15:01	17798	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
DK463788	2009-07-08 17:15:34	98062	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463788	2009-07-08 17:15:34	1938	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463788	2009-07-08 17:15:34	5000	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
FR110762	2009-07-08 17:16:11	4709	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110762	2009-07-08 17:16:11	1724	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110762	2009-07-08 17:16:11	17798	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110762	2009-07-08 17:16:11	769	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110763	2009-07-08 17:16:12	7000	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110763	2009-07-08 17:16:12	125	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110763	2009-07-08 17:16:12	16587	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
FR110763	2009-07-08 17:16:12	2288	BNP Paribas Détention	BlueNext Détention	1198.0	1351.0	10	0
DK463789	2009-07-08 17:16:33	7935	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463789	2009-07-08 17:16:33	10000	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463789	2009-07-08 17:16:33	53548	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463789	2009-07-08 17:16:33	7695	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
DK463789	2009-07-08 17:16:33	15822	Unitrade	1969 - APX-ENDEX Holding B.V. P	4571.0	1969.0	3	0
FR110764	2009-07-08 17:17:21	16587	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110764	2009-07-08 17:17:21	7000	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110764	2009-07-08 17:17:21	125	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
FR110764	2009-07-08 17:17:21	2288	BlueNext Détention	TOTAL GLOBAL STEEL LTD	1351.0	1504.0	10	0
DE79454	2009-07-08 17:18:55	474	1900 - UniCredit Bank AG	Fortis Bank	1900.0	237.0	3	0
DE79454	2009-07-08 17:18:55	28601	1900 - UniCredit Bank AG	Fortis Bank	1900.0	237.0	3	0

Figure: An example of transaction log data.

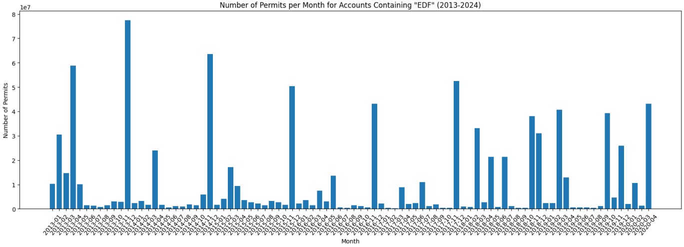

Figure: An example of reconstitution of permits detention.

Model

Research question

- If we consider a wide class of dynamic allocation processes, which ones are optimal to achieve a given expected emissions reduction over a finite time horizon at minimal expected abatement cost?

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- If we consider a wide class of dynamic allocation processes, which ones are optimal to achieve a given expected emissions reduction over a finite time horizon at minimal expected abatement cost?

Answer

- They are those which maintain the price of allowances constant.
- Optimal dynamic allocation processes compensate firms for business cycles economic shocks.

Emissions

- A regulator wishes to reduce the emissions of a set of N firms over a period of time $(0, T)$.
- Each firm i emissions follows the dynamics

$$E_t^i = \mu_i t + \sigma_i W_t^i, \quad \text{with} \quad W^i := \sqrt{1 - {k_i}^2}\,\tilde{W}^i + k_i \tilde{W}^0$$

  and $\tilde{W}_i$, $i = 0, \ldots, N$ independent, $\rho_{ij} := k_i k_j$.
- In the Business As Usual scenario (BAU), total expected emissions at time T are

$$\mathbb{E}\big[E_T\big] = N\bar{\mu}T, \quad \bar{\mu} := \frac{1}{N}\sum_{i=1}^{N} \mu_i.$$

- The regulator wishes to reduce the emissions to

$$L := \rho N \bar{\mu} T, \quad \rho \in (0, 1).$$

Dynamic allocations, bank accounts, abatement efforts and trading

- The regulator consider as possible instruments dynamic allocations of allowances.
- Denote $\tilde{A}^i_t$ the cumulative allocation process to firm i up to time t.
- The variation of the cumulative allocation $d\tilde{A}^i_t$ can be composed of, an absolutly continous part $\tilde{a}^i_t dt$, a Brownian part $\sum_{j=1}^{N} \tilde{b}^{i,j}_t \cdot dW^j_t$, and/or a singular part (jumps).

Dynamic allocations, bank accounts, abatement efforts and trading

- At time $t = 0$, the regulator opens a bank account for each firm i and credit (or debit) the account by the value A_0^i of allowances.
- The dynamics of the bank account of firm i is given by

$$dX_t^i = d\tilde{A}_t^i + \beta_t^i dt - dE_t^{i,\alpha^i}, \quad dE_t^{i,\alpha^i} = -\alpha_t^i dt + \mu_i dt + \sigma_i dW_t^i.$$

- The process α^i is the abatement effort rate.
- The process β^i is the trading rate.
- Dynamics of the bank accounts rewrites

$$dX_t^i = d\tilde{A}_t^i + \left(\alpha_t^i + \beta_t^i - \mu_i\right) dt - \sigma_i dW_t^i.$$

Firms objective

- Abatement efforts of firm i comes at a cost

$$c_i(\alpha) := \underbrace{h_i\alpha}_{\text{prop. cost}} + \underbrace{\frac{1}{2}\frac{\alpha^2}{\eta_i}}_{\text{adjustment cost}}.$$

- For a given price process of allowances P and a given dynamic allocation scheme $(\tilde{A}^i)_i$, each firm i wishes to solve

$$\inf_{\alpha^i,\beta^i} J^i(\alpha^i,\beta^i) := \mathbb{E}\Big[\int_0^T \Big(c_i(\alpha_t^i) + P_t\beta_t^i\Big)dt + \lambda\big(X_T^i\big)^2\Big],$$

and λ a parameter for the terminal bank account imbalances, reflects long-term social damages.

Market Equilibrium

- For a given allocation scheme $(\tilde{A}^i)_i$, a market equilibrium is a vector of processes $(\hat{\alpha}, \hat{\beta})$ such that

$$J^i(\hat{\alpha}^i, \hat{\beta}^i) = \inf_{\alpha^i, \beta^i} J^i(\alpha^i, \beta^i), \quad \text{and} \quad \sum_{i=1}^{N} \hat{\beta}^i_t = 0.$$

Regulator's optimisation problem

Minimise total abatement costs and terminal penalty costs while ensuring a given emissions reduction.

$$\inf_{\tilde{A}} R(\tilde{A}) := \mathbb{E}\Big[\sum_{i=1}^{N} \int_0^T c_i(\hat{\alpha}_t^i)dt + \lambda(\hat{X}_T^i)^2\Big],$$

$$\mathbb{E}\Big[\sum_{i=1}^{N} E_T^{i,\hat{\alpha}^i}\Big] = L = \rho T N \bar{\mu}.$$

Remarks

- Full observability of abatement and trading rates and of economic shocks.
- Possibility to introduce linear market frictions.
- The model is inspired from Kollenberg and Tascini (JEEM, 2016).

Some notations and useful variables

- Define the processes

$$A_t^i := \tilde{A}_t^i - \mu_i t, \quad M_t^i := \mathbb{E}_t\left[A_T^i\right],$$

resp. the net cumulative allocation A^i and the conditional expectation of the total allocation M^i.

- And also the average quantities

$$\bar{A}_t := \frac{1}{N}\sum_{i=1}^N A_t^i \quad \bar{M}_t := \frac{1}{N}\sum_{i=1}^N M_t^i = \mathbb{E}_t[\bar{A}_T], \qquad \bar{H} := \frac{1}{N}\sum_{i=1}^N \eta_i h_i,$$

$$\bar{W}_t := \frac{1}{N}\sum_{i=1}^N \sigma_i W_t^i, \quad \bar{X}_t := \frac{1}{N}\sum_{i=1}^N X_t^i.$$

Results

Market Equilibrium

(i) For a given market net allocation scheme $(A^i)_i$, the equilibrium price $\hat{P}$ is a martingale given by

$$d\hat{P}_t = -f(t)\Big(d\bar{M}_t - d\bar{W}_t\Big), \quad \hat{P}_0 = f(0)\Big(T\bar{H} - \bar{M}_0\Big),$$

$$\text{with} \quad f(t) := \frac{2\lambda}{1 + 2\lambda\bar{\eta}(T-t)}.$$

(ii) The abatement effort of firm i is unique and given by:

$$\hat{\alpha}^i_t = \eta_i\big(\hat{P}_t - h_i\big).$$

(iii) Trading rates β^i are non unique, only total trading quantities are.

- If firms expect that more allowances are going to be injected ($d\bar{M}_t > 0$), the price $\hat{P}$ decreases.
- If the economy experiences a positive shock ($d\bar{W}_t > 0$), the price increases.

Why?

- Take firm i criteria

$$J^i(\alpha^i,\beta^i) := \mathbb{E}\Big[\int_0^T \Big(h_i\alpha_t^i + \frac{1}{2}\frac{(\alpha_t^i)^2}{\eta_i} + P_t\beta_t^i\Big)dt + \lambda\big(X_T^i\big)^2\Big].$$

- First-order conditions w.r.t. α^i and β^i are resp.

$$\underbrace{h_i + \frac{1}{\eta_i}\alpha_t^i}_{\text{marginal cost}} + \underbrace{2\lambda\mathbb{E}_t\big[X_T^i\big]}_{\text{marginal penalty}} = 0, \quad P_t + 2\lambda\mathbb{E}_t[X_T^i] = 0.$$

- Thus, the price satisfies

$$P_t = -\frac{2\lambda}{N}\sum_{i=1}^{N}\mathbb{E}_t\big[X_T^i\big] = -2\lambda\mathbb{E}_t\big[\bar{X}_T\big].$$

- And the α^i are martingales satisfying

$$\alpha_t^i = \eta_i(P_t - h_i), \quad \bar{\alpha}_t = \bar{\eta}P_t - \bar{H}, \quad d\bar{\alpha}_t = \bar{\eta}dP_t.$$

Why? (cont.)

- Since $\bar{\alpha}$ is a martingale,

$$\mathbb{E}_t\left[\bar{X}_T\right] = \mathbb{E}_t\left[\bar{A}_T + \int_0^T \bar{\alpha}_s ds + \underbrace{\int_0^T \bar{\beta}_s ds}_{=0} - \bar{W}_T\right]$$

$$= \bar{M}_t + \int_0^t \bar{\alpha}_s ds + (T-t)\bar{\alpha}_t - \bar{W}_t.$$

- Thus,

$$dP_t = -2\lambda d\mathbb{E}_t\left[\bar{X}_T\right] = -2\lambda\Big[d\bar{M}_t + (T-t)d\bar{\alpha}_t - d\bar{W}_t\Big].$$

- Subtitution of $d\bar{\alpha}_t = \bar{\eta} dP_t$ provides

$$dP_t = -\frac{2\lambda}{1 + 2\lambda\bar{\eta}(T-t)}\Big[d\bar{M}_t - d\bar{W}_t\Big].$$

Consequences for optimal regulation

- Total expected emissions only depend on average effort rate $\bar{\alpha}$ and since it is a martingale, we have

$$\mathbb{E}\left[N\bar{E}_T\right] = NT\left(\bar{\mu} - \bar{\alpha}_0\right) = NT\left(\bar{\mu} - \bar{\eta}\hat{P}_0 + \bar{H}\right).$$

- To achieve a reduction by a factor ρ it should hold that

$$\hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}.$$

Comment

The average price is made of two components.

- The average of the linear part of the marginal abatement cost
- The term taking into account the adjustment cost, the growth rate of emissions and the ambition of the regulation.

Consequences for optimal regulation

- The expression of $\hat{P}_0$, i.e.

$$\hat{P}_0 = f(0)\Big(T\bar{H} - \bar{M}_0\Big)$$

says that it is fully determined by $\bar{M}_0$.

- Thus, to achieve a reduction by a factor ρ, one should pick $\bar{M}_0$ such that:

$$\bar{M}_0 = -\frac{1}{2\lambda\bar{\eta}}\Big[\bar{H} + \big(1 + 2\lambda\bar{\eta}T\big)(1-\rho)\bar{\mu}\Big] =: \ell(\rho) < 0.$$

Comment

- Suppose that the regulator does not want to add or withdraw on average allowances. It means

$$\bar{M}_0 = \bar{A}_0 + \underbrace{\mathbb{E}\Big[\int_0^T d\bar{A}_t\Big]}_{=0} = \bar{A}_0 < 0.$$

- On average, the bank accounts should be endowed with a debt.

Rephrasing the regulator's optimisation problem

Recall the regulator's optimisation problem is

$$\inf_{\tilde{A}} R(\tilde{A}) := \mathbb{E}\Big[\sum_{i=1}^{N}\int_0^T c_i(\hat{\alpha}_t^i)dt + \lambda(\hat{X}_T^i)^2\Big], \quad \mathbb{E}\Big[\sum_{i=1}^{N} E_T^{i,\hat{\alpha}^i}\Big] = L = \rho T N\bar{\mu}.$$

Using the fact that

$$\hat{P}_T = -2\lambda\mathbb{E}_T\left[X_T^i\right] = -2\lambda X_T^i$$

The regulator problems can be written

$$\inf_{A} \mathbb{E}\Big[\sum_{i=1}^{N}\int_0^T \Big(h_i\eta_i(\hat{P}_t - h_i) + \frac{1}{2}\eta_i(\hat{P}_t - h_i)^2\Big)dt + \frac{(\hat{P}_T)^2}{4\lambda}\Big]$$

$$d\hat{P}_t = -f(t)(d\bar{M}_t - d\bar{W}_t), \quad \hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}, \quad \bar{M}_0 = \ell(\rho).$$

Remarks

- The expected price $\mathbb{E}\big[\hat{P}_t\big] = \hat{P}_0$ is determined by the level of reduction.
- Further, $\mathbb{E}\big[\hat{P}_t^2\big] = \hat{P}_0^2 + \mathbb{E}\big[\langle \hat{P} \rangle_t\big]$.
- Thus, the regulator's minimisation problem boils down to minimising the quadratic variation of the price, i.e., its volatility.
- The M^i are martingales. They can be written

$$M_t^i = M_0^i + \int_0^T \gamma_t^i \cdot dW_t, \quad \gamma^i := (\gamma^{i,k}).$$

- We just need to find γ^i so that $\langle \bar{M} - \bar{W} \rangle = 0$.
- One possibilitity is simply $\gamma_t^i = \sigma^i W_t^i$, i.e.

$$M_t^i = M_0^i + \sigma^i W_t^i.$$

- And thus,

$$A_t^i = M_0^i + \sigma^i W_t^i.$$

  satisfies the condition $M_t^i = \mathbb{E}_t\big[A_T^i\big]$.

Optimal regulations

(i) The solutions to the regulator optimisation problem are non-unique and characterised by the minimisation of the price volatility and the condition that $\bar{M}_0 = \ell(\rho)$.

(ii) The cumulative allocations given by

$$\tilde{A}^i_t = \ell(\rho) + \mu_i t + \sigma_i W^i_t, \quad i = 1, \dots, N,$$

form a solution.

(iii) The equilibrium price and abatement efforts are constant given by

$$\hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}, \quad \hat{\alpha}^i_0 = \eta_i(\hat{P}_0 - h_i).$$

Comment

The regulator provides an equal debt on all firms and compensates each firm from the emission trend of the BAU and of the economic shocks that affect it.

Where is the benefit
of a dynamic allocation scheme
compared to a simple static initial allocation?

Static allocation scheme (EU TS Phase I and II)

- The static allocation scheme corresponds to

$$\tilde{A}_0^i = A_0^i = \ell(\rho), \quad d\tilde{A}_t^i = \mu_i dt + dA_t^i = 0, \quad 0 < t \leq T,$$

- For sake of computation, suppose all firms endure the same adjustement cost parameter $\eta_i = \eta$.
- Denote Δ^{stat} the difference between the social cost with a static allocation and the social cost under an optimal dynamic allocation.

Static allocation scheme (EU TS Phase I and II)

- We have

$$\Delta^{\text{stat}} := \frac{N\zeta^2}{2\eta} \ln \Big[1 + 2\lambda\eta T\Big], \qquad N^2\zeta^2 := \sum_{i=1}^{N} \sigma_i^2 + 2\sum_{i<j} \rho_{ij}\sigma_i\sigma_j.$$

- In the presence of uncertainty ($\sigma > 0$) or irreversibility (η small), there is a benefit from dynamic allocation.
- Suppose there are N firms with identical σ_i and k_i. Denote $\bar{\rho}$ the common correlation. Here, $N^2\zeta^2 = N\bar{\sigma}^2 + \bar{\rho}\bar{\sigma}N(N-1)$.
- Hence, if there is a common noise, when $N \to \infty$, the per unit difference cost Δ^{stat}/N admits a finite limit, making also dynamic schemes beneficial.

- Dynamic allocation provides insurance to firms from common economic uncertainty that induces costly adjustment.

Illustration of the dynamics

- In $T = 5$ years, in a market of $N = 6$ aggregated sectors,
- the regulator wants to reduce the emissions by 20%, i.e. $\rho = 0.8$,
- in a market where the average growth rate of emissions is $N\bar{\mu} = 2$ Gt/year,
- with a volatility of $\sigma_i = 0.2/\sqrt{N}$ Gt/year and per firm,
- and average abatement cost $\bar{h} = 25$ €/t,
- and equal adjustment cost $\eta = 6\,10^8$ t^2/€ (after Gollier (2020)),
- and a equal dependence on the common shocks of $k_i = 0.9$
- and a terminal penalty parameter of $\lambda = 1.25\,10^{-6}$ €/t^2.

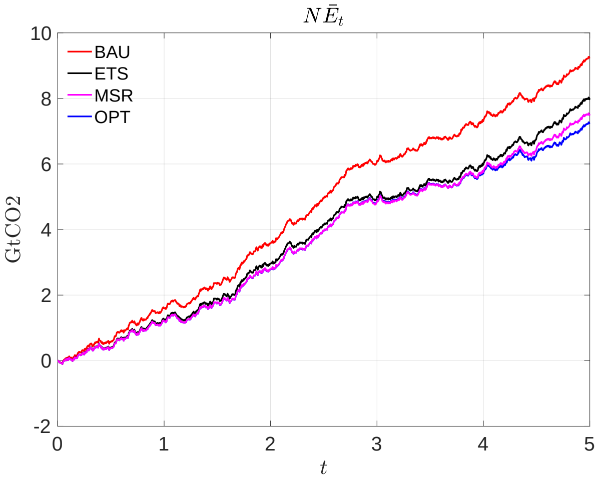


Figure: Total emission

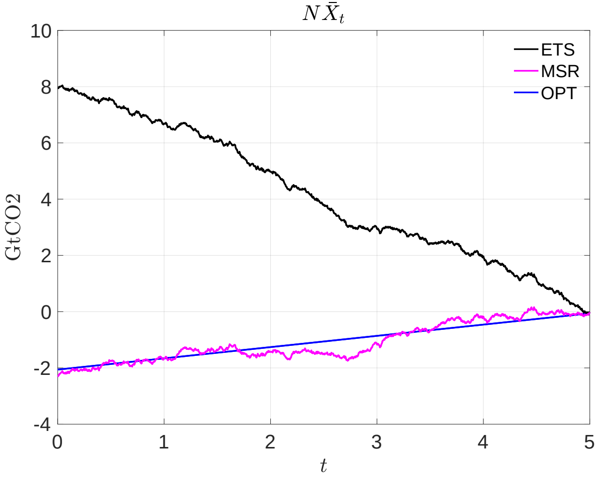


Figure: Total bank accounts

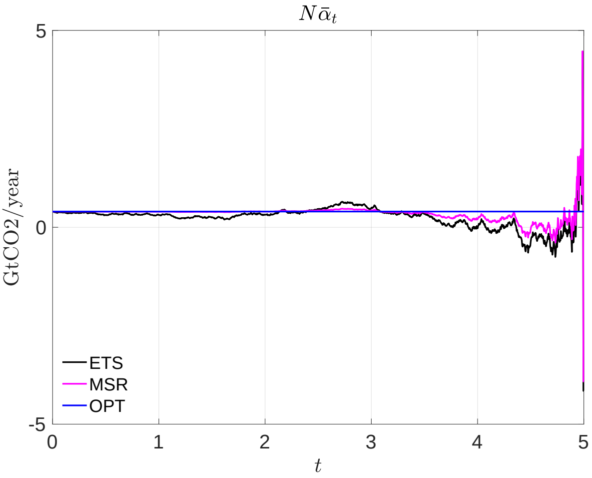


Figure: average abatement effort

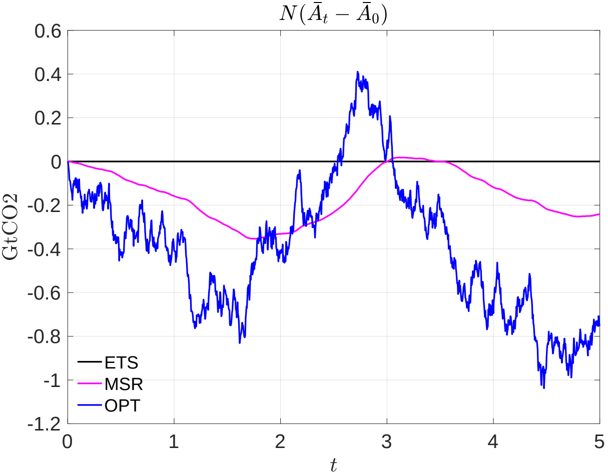


Figure: Net allocation minus initial allocation

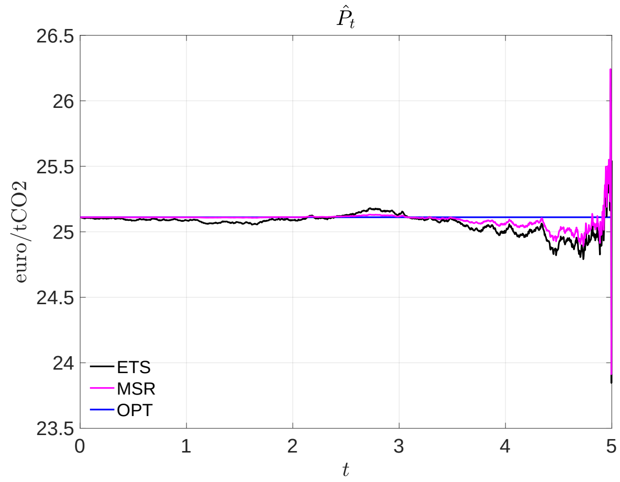


Figure: Equilibrium market price

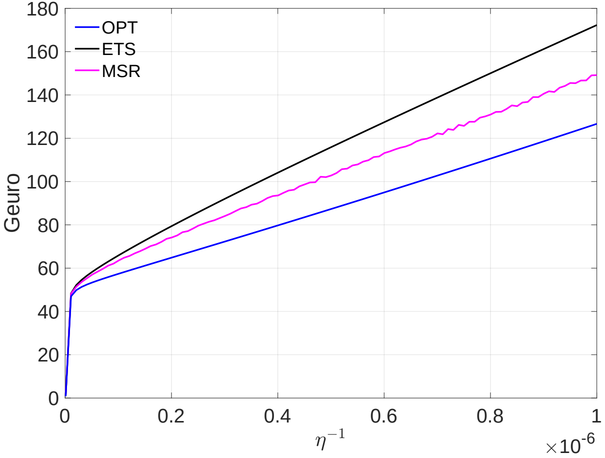


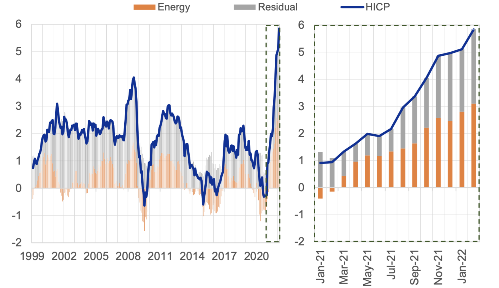
Figure: Social cost as a function of η^{-1}

Carbon emissions market and inflation

Maria Arduca Sara Biagini Luca taschini

Carbonflation

- Climate change and climate policies are likely to affect inflation
- Fossilflation to *blame* for much of recent strong increase in EU inflation.
- In February 2022, energy accounted for more than 50% of headline inflation in the euro area, mainly reflecting the sharp increases in oil and gas prices



Source: Eurostat and ECB staff calculations.
Last observation: February 2022.

Why is this a potential problem problem?

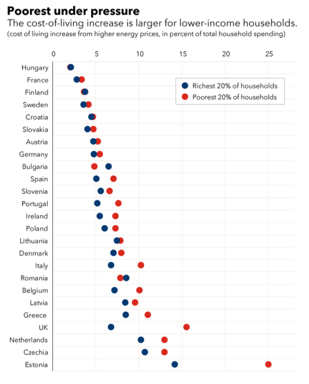


Figure: Impacts of the energy crisis across European countries (IMF report).

- Upcoming of ETS 2 for heating and road transportation. Already in JOEU in May 2023, smooth start in 2024 and full application in 2027.
- Expected direct impact on consumers
- Special mechanisms created to avoid carbonflation: non fungibility of EUA on ETS-1 and on ETS-2, special market stability reserve for ETS-2, special breaking mechanism in case of gaz price increase.

Research question

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Results

- At equilibrium, we exhibit a simple linear model linking emission target reduction and induced inflation
- We study the effect of a reduction/increase of the carbon emission reduction horizon
- We find that it has little effect on induced total inflation in EU (result which is inline with current econometric estimates)

Model & Results

Production, abatement and trade

- The dynamics of the emissions under abatement effort α^i, production decision q^i of firm i with emission intensity of production γ_i at time t is given by

$$dE^i_t = \left(\gamma_i q^i_t - \alpha^i_t\right) dt + \sigma_i dW^i_t,$$

- The emissions imbalance at the compliance date T is

$$X^i_T = E^i_T - \int_0^T \beta^i_t \, dt - Y^i_T,$$

where β^i is the trading rate of firm i Y^i_T is the cumulated amount of allowances provided from the regulator.

Production levels in the absence of carbon market

- Firm i optimizes the profit:

$$\inf_q c_i(q) - S_i(q)q \quad S_i(q) := a_i - b_i q,$$
$$c_i(q) := \kappa_i(q - \tilde{q}_i) + \frac{1}{2}\delta_i(q - \tilde{q}_i)^2$$

- Optimal production level is $q^*(\tilde{q}_i)$ and we chose $\tilde{q}_i$ such that $q^*(\tilde{q}_i) = \tilde{q}_i$, so that the pre-regulation reference production cost is zero. It leads to:

$$\tilde{q}_i = \frac{a_i - \kappa_i}{\delta_i + 2b_i}.$$

In the Business As Usual (BAU), we denote

$$\mu_i := \gamma_i \tilde{q}_i, \quad \bar{\mu}_b := \frac{1}{N}\sum_{i=1}^{N} \mu_i,$$

the emission drift of firm i under BAU, and the average drift in emissions across the entire economy.

Firm's optimisation problem in the presence of carbon market

- Given a cumulative allocation Y^i_T, firm i solves

$$\inf_{q^i,\alpha^i,\beta^i} J^i(q^i,\alpha^i,\beta^i) := \mathbb{E}\Big[\int_0^T \Big(c_i(q^i_t) + g_i(\alpha^i_t) + \beta^i_t P_t - S_i(q^i_t)q^i_t\Big)dt + \lambda(X^i_T)^2\Big]$$

  where $g_i(\alpha) := h_i\alpha + \frac{1}{2\eta_i}\alpha^2$ is the abatement cost function of firm i.
- P_t is the carbon market price to be determined at equilibrium by the usual market clearing condition.
- Terminal imbalance X^i_T is penalise with a quadratic cost mostly for tractability reasons (possible extension to perfect zero imbalance).

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$$\text{Optimal production} \quad \hat{q}^i_t = \tilde{q}_i - \frac{\gamma_i}{\delta_i + 2b_i} P_t,$$
$$\text{Optimal abatement} \quad \hat{\alpha}^i_t = \eta_i (P_t - h_i).$$

Remark: only cumulated optimal trading is uniquely defined.

Market Equilibrium

- For a given allocation scheme $(Y^i)_i$, a market equilibrium is a vector of processes $(\hat{q}, \hat{\alpha}, \hat{\beta})$ such that

$$J^i(\hat{q}_i, \hat{\alpha}^i, \hat{\beta}^i) = \inf_{q^i, \alpha^i, \beta^i} J^i(q^i, \alpha^i, \beta^i), \quad \text{and} \quad \sum_{i=1}^{N} \hat{\beta}^i_t = 0.$$

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Equilibrium price

It is unique given by

$$d\hat{P}_t = f(t)(d\bar{W}_t - d\bar{M}_t), \quad \hat{P}_0 = f(0)\Big[(\bar{H} + \bar{\mu}_b)T - \mathbb{E}[\bar{Y}_T]\Big],$$

$$\text{with} \quad f(t) := \frac{2\lambda}{1 + 2\lambda(\bar{\eta} + \bar{\psi})(T - t)} \quad \bar{H} = \frac{1}{N}\sum_i \eta_i h_i.$$

where $\bar{M}_t = \mathbb{E}_t[\bar{Y}_T]$ and $\bar{W}$ the average shock.

Emission reduction target

- Assume the regulator wishes to achieve net-zero emission at the horizon T.
- In our context, it translates into

$$\mathbb{E}[\bar{\mu}_T] = 0, \quad \bar{\mu}_T := \frac{1}{N}\sum_i \mu^i_T, \quad \mu^i_T := \gamma_i \hat{q}^i_T - \hat{\alpha}^i_T.$$

- We know the optimal production and abatement as functions of P and thus, we deduce that

$$\mathbb{E}[\hat{P}_T] = \hat{P}_0 = \frac{1}{\bar{\phi}}(\bar{\mu}_b + \bar{H}).$$

and using the equilibrium result on P we can deduce

$$\hat{P}_0 = f(0)\Big[(\bar{H} + \bar{\mu}_b)T - \mathbb{E}[\bar{Y}_T]\Big],$$

the total expected allocation that is needed.

Inflation concern

- For a given allocation $(Y^i)_i$ and induced carbon price $\hat{P}$, we consider the price index $\hat{\pi}$ of a weighted sum of the produced goods prices S_i:

$$\hat{\pi}_t := \sum_{i=1}^{N} w_i S_i(\hat{q}_t^i), \quad \pi_b := \sum_{i=1}^{N} w_i S_i(\tilde{q}_i), \quad w_i \in (0,1), \quad \sum_i w_i = 1,$$

- The policy-induced inflation over BAU is the difference:

$$\hat{\pi}_T - \pi_b = \sum_{i=1}^{N} w_i (S_i(\hat{q}_T^i) - S_i(\tilde{q}_i)) = \underbrace{\sum_{i=1}^{N} \frac{b_i}{\delta_i + 2b_i} \gamma_i w_i}_{\bar{\omega}} \hat{P}_T$$

- Expected average inflation rate over the period T

$$I := \frac{\mathbb{E}[\hat{\pi}_T] - \pi_b}{T \pi_b} = \frac{\bar{\omega}}{T \pi_b} \hat{P}_0$$

Resume of the situation

- To achieve net zero by T, we have

$$\hat{P}_0 = f(0)\Big[(\bar{H} + \bar{\mu}_b)T - \mathbb{E}[\bar{Y}_T]\Big], \quad \hat{\imath}_T := \frac{\mathbb{E}[\hat{\pi}_T] - \pi_b}{\pi_b} = \frac{\bar{\omega}}{\pi_b}\hat{P}_0$$

- Total inflation does not depend on T, only expected average rate of inflation.
- If social cost of inflation depends on total inflation, then time horizon T does not matter here.
- If average growth rate of inflation is the driver of wealth loss, then it matters.
- Yes, but how much?

Environmental concern meets monetary regulation

- Balancing environmental concerns and inflation:

$$\inf_{(Y^i)_i} J^{\mathrm{R}}(Y) := \sum_{i=1}^{N} J^i(\hat{\alpha}^i, \hat{q}^i, \hat{\beta}^i) + \mathbb{E}\Big[\ell(\bar{\mu}_T - \theta)\Big] + \mathbb{E}\Big[\varphi\left(\hat{\imath}_T - \nu\right)\Big]$$

- ℓ is a convex penalization on the deviation of the realized average emissions drift $\bar{\mu}_T$ from the target θ;
- φ is a convex penalization on excess average growth rate of inflation compared to a given target ν (e.g. $\nu = 1\%$/year).

Solution to the regulator's problem

- Under optimal allocation process, the equilibrium is constant, $\hat{P}_t = P^*$.
- The optimal allocation process Y is optimal iff it satisfies

$$d\bar{M}_t^* = d\bar{W}_t, \quad \bar{M}_0^* = T\bar{H} - P^*\Big(\frac{1}{2\lambda} + \bar{\phi}T\Big),$$

  where $\bar{M}_t^* = \mathbb{E}_t[\bar{Y}_T^*]$.
- Induced optimal production level and abatement effort are constant

$$\alpha_t^{i,*} = \eta_i(P^* - h_i) \qquad q_t^{i,*} = \tilde{q}_i - \frac{\psi_i}{\gamma_i}P^*.$$

  and average emission rate at terminal time and average growth rate of inflation are

$$\bar{\mu}_T^* = \bar{\mu}_b - \bar{\psi}P^*, \quad \hat{\imath}_T^* = \frac{\bar{w}}{T\pi_b}P^*.$$

Case of quadratic penalisation

- When $\ell(x) := \lambda_e x^2$ and $\varphi(x) := \lambda_\imath x^2$, our model reduces to the relations

$$\mathbb{E}[\bar{\mu}_T] = \bar{\mu}_b + \bar{H} - \bar{\phi} P^*, \quad \hat{\imath}^*_T = \bar{\omega}\frac{P^*}{\pi_b}, \quad \text{emissions and inflation}$$

$$P^* = \frac{\bar{\phi}(\bar{\mu}_b + \bar{H} - \theta)\lambda_e + \lambda_\imath \frac{\bar{\omega}\nu}{T\pi}}{NT(\frac{1}{2}\bar{\phi} + \frac{1}{4\lambda T}) + \bar{\phi}^2\lambda_e + \frac{\bar{\omega}^2}{T^2\pi^2}\lambda_\imath}, \quad \text{carbon price}$$

$$s(P^*) - s(0) = NT\left(\frac{1}{2}\bar{\phi} + \frac{1}{4\lambda T}\right)(P^*)^2$$

$$+ \lambda_e\left(\bar{\mu}_b - \bar{\phi}P^* + \bar{H} - \theta\right)^2 + \lambda_\imath\left(\frac{\bar{\omega}P^*}{T\pi} - \nu\right)^2, \quad \text{social cost.}$$

Evidence of the effect of GHGs on Inflation

- Theoretical studies tend to indicate significant effects on consumer price inflation (McKibbin et al., 2017; Goulder and Hafstead, 2018).
- Some empirical evidences
  - Konradt and Weder di Mauro (2021): weak effect for EU and CAN
  - Kaenzig (2021): positive effect on energy and consumer prices
  - Moessner (2022): US$10 increase per ton of CO_2 increases energy inflation by 0.8pp and headline inflation by 0.08p
  - Coenen et al (2023): increase energy inflation by 0.2pp in the course of 2023
  - Konradt and McGregor and Toscani (2024): price increase from 40 Euro per ton of CO2 in 2021 to 150 Euro by 2030 could raise annual Euro area inflation by between 0.2 and 0.4 percentage points.

Model calibration

T	$\bar{\mu}_b$	$\bar{H}$	$\bar{\phi}$	$\bar{\omega}$	$\lambda_\imath$	λ_e	λ
y	Gt/y	Gt€^{2}/y	Mt/€	€$^{-1}$	G€/(%/y)^2	€/(tCO_2)^2	€/ton^2
25	1.5	15	6.25	0.175	750	$2\,10^{-3}$	$1.25\,10^{-6}$

Table: Parameters value of the model

Anticipating net zero

(a) (b) (c)

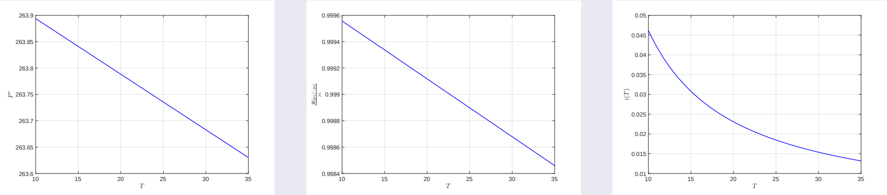

Figure: As a function of the period T (a) optimal carbon emission equilibrium price P^* (b) percentage of emission rate reduction $\frac{\mathbb{E}[\bar{\mu}_T]-\bar{\mu}_b}{\bar{\mu}_b}$ (c) average inflation rate $\imath_T$.

The equilibrium is not affected much by an anticipation of the net-zero time horizon, as well as the percentage of emission rate reduction. The average annual inflation rate is impacted by 0.04 bp (it moves from 1% to 1.04%).

Social costs of hastening decarbonation

(a) (b) (c) (d)

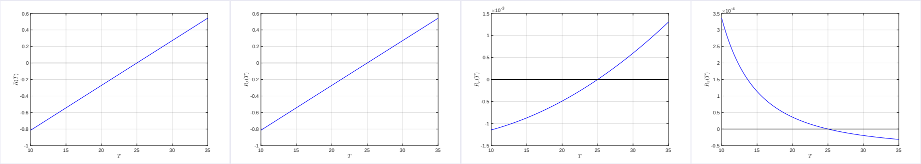

Figure: As a function of the period T (a) $R(T)$ (b) $R_\lambda(T)$ (c) $R_\mu(T)$ (d) $R_\pi(T)$.

The increase in social cost from the inflation component $(R_\pi(T))$ does not hinder the benefits from the other components. The net effect of hastening net-zero emission is positive because of the large value of carbon emissions damages.

Conclusions & Perspectives

More research questions

- Relaxing hypothesis
- Extension of EU ETS 2
- Detention/retention of emissions by consumers (Homaio initiative `https://www.homaio.com/fr`)